**1.** Consider the following claim about positive real numbers $x$ and $y$:

> If $x$ and $y$ each round to 2 to the nearest integer, then $x + y$ rounds to 4 to the nearest integer.

Exact half-integers are rounded upwards. Which of the following is a counterexample to the claim?

**A** $x = 1.4, \quad y = 2.3$

**B** $x = 1.7, \quad y = 2.1$

**C** $x = 1.8, \quad y = 2.3$

**D** $x = 1.6, \quad y = 1.8$

**E** $x = 2.4, \quad y = 2.6$

**2.** The real numbers $a$ and $b$ satisfy

$$3a + 2b = 7, \qquad 2a + 3b = 8.$$

What is the value of $a^2 - b^2$?

**A** $-3$

**B** $-1$

**C** 1

**D** 3

**E** 9

**3.** The graph

$$y = \frac{2x + 1}{x - 3} \qquad (x \neq 3)$$

has no point in common with the horizontal line $y = k$.

What is the value of $k$?

**A** $-\frac{1}{2}$

**B** $-\frac{1}{3}$

**C** $\frac{1}{2}$

**D** 2

**E** 3

**4.** For a particular real number $x$, consider the statement

$$S: \quad \text{If } x^2 < 9, \text{ then } x > 1.$$

Which of the following statements about $x$ is logically equivalent to $S$?

**A** $-3 < x \leq 1$

**B** $x > 1$

**C** $x \leq -3$ or $x > 1$

**D** $x < -3$ or $x \geq 1$

**E** $1 < x < 3$

**5.** For how many positive integer values of $k$ does the equation

$$9^x - 8 \cdot 3^x + k = 0$$

have two distinct real solutions for $x$, one positive and the other negative?

**A** 6

**B** 7

**C** 8

**D** 15

**E** 16

**6.** Let $a$ and $b$ be real numbers. Consider the statements

$P$: $a \geq |b|$.

$Q$: $a(x^2 + 1) + 2bx \geq 0$ for every real number $x$.

Which of the following correctly describes $P$ as a condition for $Q$?

**A** $P$ is necessary but not sufficient for $Q$.

**B** $P$ is both necessary and sufficient for $Q$.

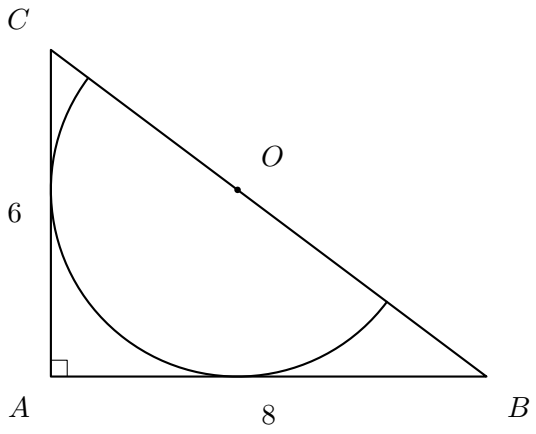
**C** $P$ is neither necessary nor sufficient for $Q$.

**D** $P$ is sufficient but not necessary for $Q$.

**7.** Triangle $ABC$ is right-angled at $A$, with $AB = 8$ and $AC = 6$.

A semicircle with centre $O$ lies inside the triangle. Its diameter lies on $BC$, and its arc is tangent to both $AB$ and $AC$, as shown.

What is the radius of the semicircle?




**A** 2

**B** $\frac{12}{5}$

**C** $\frac{24}{7}$

**D** $\frac{24}{5}$

**E** $\frac{48}{7}$

**8.** A list consists of five distinct positive real numbers. One number is removed, leaving a list of four numbers.

Consider the statements

$P$: The number removed equals the mean of the original list.

$Q$: Both the mean and the median are unchanged by the removal.

Which of the following is correct?

**A** $P$ is sufficient but not necessary for $Q$.

**B** $P$ is both necessary and sufficient for $Q$.

**C** $P$ is necessary but not sufficient for $Q$.

**D** $P$ is neither necessary nor sufficient for $Q$.

**9.** A sequence is defined by

$$u_1 = 3, \qquad u_{n+1} = u_n + (-1)^{n+1} n \quad \text{for } n \geq 1.$$

What is the sum of all the positive terms among $u_1, u_2, \ldots, u_{20}$?

**A** 70

**B** 85

**C** 88

**D** 91

**E** 112

**10.** Let $a$ and $b$ be positive real numbers satisfying $a + b + ab = 3$, and write $s = a + b$. A student gives the following proof that $ab \leq 1$.

I. Since $(a - b)^2 \geq 0$, we have $s^2 \geq 4ab$.

II. Substituting $ab = 3 - s$ gives $s^2 + 4s - 12 \geq 0$.

III. Hence $(s - 2)(s + 6) \geq 0$, so $s \leq -6$ or $s \geq 2$.

IV. Since $s > 0$, it follows that $s \geq 2$.

V. Therefore $ab = 3 - s \leq 1$.

Which of the following is correct?

**A** The first error is in line I.
**B** The first error is in line II.
**C** The first error is in line III.
**D** The first error is in line IV.
**E** The first error is in line V.
**F** All five lines I–V are valid.

**11.** Three fair six-sided dice, each numbered 1 to 6, are rolled independently.

Given that the largest of the three scores occurs at least twice, what is the probability that the sum of the three scores is even?

**A** $\frac{2}{5}$

**B** $\frac{7}{17}$

**C** $\frac{1}{2}$

**D** $\frac{10}{17}$

**E** $\frac{3}{7}$

**12.** A function $f$ is defined for every real number. Consider the statement:

> For every real number $k > 0$, there exist distinct real numbers $x$ and $y$ such that $f(x) = f(y) = k$ and $x + y < 0$.

Which of the following is the logical negation of this statement?

**A** For every $k > 0$, every pair of distinct real numbers $x, y$ satisfying $f(x) = f(y) = k$ has $x+y \geq 0$.

**B** There exist $k > 0$ and distinct real numbers $x, y$ such that $f(x) = f(y) = k$ and $x + y \geq 0$.

**C** There exists $k > 0$ such that every pair of distinct real numbers $x, y$ satisfying $f(x) = f(y) = k$ has $x + y > 0$.

**D** There exists $k > 0$ such that, for every pair of distinct real numbers $x, y$, either $f(x) \neq k$ or $f(y) \neq k$ or $x + y \geq 0$.

**E** There exist distinct real numbers $x, y$ such that, for every $k > 0$, either $f(x) \neq k$ or $f(y) \neq k$ or $x + y \geq 0$.

**13.** A particle moves along a straight line. At time $t$ seconds its coordinate, in metres, is

$$s(t) = \frac{t^2 - 5t + 4}{t}, \qquad 1 \leq t \leq 4.$$

What is its average speed, in metres per second, over the time from $t = 1$ to $t = 4$?

**A** $0$

**B** $\frac{1}{3}$

**C** $\frac{1}{2}$

**D** $\frac{2}{3}$

**E** $\frac{3}{4}$

**14.** Let $a$ and $b$ be positive integers. Consider the statements

$P$: $a$ and $b$ are consecutive integers.

$Q$: The highest common factor of $a + b$ and $a^2 + b^2$ is 1.

Which of the following is correct?

**A** $P$ is sufficient but not necessary for $Q$.

**B** $P$ is neither necessary nor sufficient for $Q$.

**C** $P$ is both necessary and sufficient for $Q$.

**D** $P$ is necessary but not sufficient for $Q$.

**15.** A triangle has vertices $(0, 0)$, $(6, 0)$ and $(0, 6)$. A straight line $\ell$ passes through the interior of the triangle.

Consider the statements

$P$: The line $\ell$ passes through $(2, 2)$.

$Q$: The line $\ell$ divides the triangle into two regions of equal area.

Which statement correctly describes $P$ as a condition for $Q$?

**A** $P$ is both necessary and sufficient for $Q$

**B** $P$ is sufficient but not necessary for $Q$

**C** $P$ is neither necessary nor sufficient for $Q$

**D** $P$ is necessary but not sufficient for $Q$

**16.** For which real values of $k$ does the equation

$$\sin^2\theta - (k+1)\sin\theta + \frac{k^2}{4} = 0$$

have exactly four distinct solutions satisfying $0 \leq \theta \leq 2\pi$?

**A** $-\frac{1}{2} < k < 0$

**B** $-\frac{1}{2} < k \leq 0$

**C** $-\frac{1}{2} \leq k \leq 0$

**D** $-\frac{1}{2} < k < 1$

**E** $k < 0$

**F** $0 \leq k < 4$

**17.** A finite set of points in the plane is not contained in a straight line. The following proof shows that some straight line contains exactly two of the points.

Among all choices of a point $P$ from the set and a line $\ell$ through two other points, with $P$ not on $\ell$, choose one for which the perpendicular distance $PH$ is least. Here $H$ is the perpendicular foot on $\ell$.

Suppose that $\ell$ contains at least three points of the set.

*[Missing argument]*

This gives a smaller positive distance of the same kind, a contradiction. Hence $\ell$ contains exactly two points.

Which option correctly supplies the missing argument? In each option, $A, B$ must be points of the set on $\ell$.

**A** Choose points $A, B$ on opposite sides of $H$. The distance from $A$ to the line $PB$ is $PH \cdot AB/PB < PH$, since $AB < PB$.

**B** Choose points $A, B$ on one closed ray from $H$, with $HA < HB$. The distance from $B$ to the line $PA$ is $PH \cdot AB/PA < PH$, since $AB < PA$.

**C** Choose points $A, B$ on one closed ray from $H$, with $HA < HB$. The distance from $A$ to the line $PB$ is $PH \cdot HA/PB < PH$, since $HA < PB$.

**D** Choose points $A, B$ on one closed ray from $H$, with $HA < HB$. The distance from $A$ to the line $PB$ is $PH \cdot AB/PB < PH$, since $AB \leq HB < PB$.

**E** Choose $A$ nearest to $H$ and $B$ farthest from $H$ among the points on $\ell$. The distance from $A$ to the line $PB$ is $PH \cdot AB/PB < PH$, since $AB < PB$.

**18.** Two lists of real numbers have numbers of entries in the ratio 3 : 2. Their means are 20 and 12.5, respectively.

One entry is moved from the first list to the second. The means of the two lists then become 21 and 13, respectively.

How many entries are there in the two lists altogether?

**A** 2

**B** 4

**C** 5

**D** 6

**E** 10

**19.** Four distinct real numbers $a, b, c, d$ satisfy

$$a + b + c + d = 0, \qquad a^3 + b^3 + c^3 + d^3 = 0.$$

Which of the following statements must be true?

I. The median of the four numbers is 0.

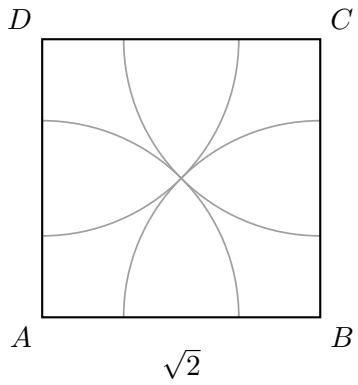
II. The product $abcd$ is positive.

III. When written in increasing order, the four numbers form an arithmetic progression.

**A** none of them
**B** I only
**C** II only
**D** III only
**E** I and II only
**F** I and III only
**G** II and III only
**H** I, II and III

**20.** A square $ABCD$ has side length $\sqrt{2}$. Four circles, each of radius 1, are drawn with centres at $A$, $B$, $C$ and $D$.

What is the area of the part of the square that lies inside exactly one of the four circles?




**A** $\dfrac{4-\pi}{2}$

**B** $\pi - 2$

**C** $4 - \pi$

**D** $1 - \dfrac{\pi}{4}$

**E** $2\pi - 4$

**F** $\pi$