1. The graph of $y = x^2 + px + q$, where p and q are real constants, is translated by 1 unit in the positive x-direction. The resulting graph has equation

$$y = x^2 + qx + p.$$

What is the value of p?

A -3

B -2

C -1

D 0

E 1

2. The derivative f' of a function f is continuous for $1 \leq x \leq 6$, and

$$\int_1^3 f'(x)\,\mathrm{d}x = 8, \qquad \int_3^6 f'(x)\,\mathrm{d}x = -3.$$

What is the gradient of the straight line joining $(1, f(1))$ and $(6, f(6))$?

A -1

B $\dfrac{5}{6}$

C 1

D $\dfrac{11}{5}$

E 5

3. The first nine terms $u_1, u_2, \ldots, u_9$ of an arithmetic progression are all positive. It is given that

$$u_2 u_8 = 40 \qquad \text{and} \qquad u_4 u_6 = 48.$$

What is $u_1 + u_2 + \cdots + u_9$?

A 7

B 49

C $36\sqrt{3}$

D 63

E 126

4. Packet A contains 25% more rice than packet B, and its total price is 10% higher than the total price of packet B.

The price per 100 grams of rice in packet A is 44 pence.

What is the price per 100 grams of rice in packet B, in pence?

A 40

B 44

C 48.4

D 50

E 55

5. Triangle ABC has

$$\angle BAC = 120^\circ, \qquad BC = 2\sqrt{7}, \qquad AB = AC + 2.$$

What is the length of AB?

A 2

B 4

C 6

D $2\sqrt{7}$

E $1 + \sqrt{13}$

6. A circle passes through $O = (0, 0)$ and $P = (4, 2)$. Its centre lies on the line $y = 2x$.

The circle meets the x-axis again at Q. What is the equation of the tangent to the circle at Q?

A $y = 2x - 5$

B $y = -2x + 5$

C $y = -\frac{1}{2}x + \frac{5}{4}$

D $y = \frac{1}{2}x - \frac{5}{2}$

E $y = \frac{1}{2}x - \frac{5}{4}$

7. Real numbers x, y and z satisfy

$$xy + x + y = 5, \qquad yz + y + z = 11, \qquad zx + z + x = 7.$$

What is the sum of all the distinct possible values of xyz?

A -60

B -54

C -52

D 0

E 6

F 54

8. A list of 15 real numbers has median 7 and range 6.

A new list is formed by replacing every number x in the original list by $20 - 2x$.

What is the sum of the median and the range of the new list?

A -6

B 6

C 12

D 18

E 38

F 46

9. The real numbers x and y satisfy

$$2^{x+y} = 16, \qquad 4^x + 4^y = 40.$$

What is $|x - y|$?

A $\frac{1}{2}$
B 1
C $\frac{3}{2}$
D 2
E $\frac{5}{2}$
F 4

10. A point P lies on the curve

$$y = x^2 + 3 + \frac{1}{x}, \qquad x > 0.$$

The origin is O. What is the least possible gradient of the straight line OP?

A -3

B 2

C $2\sqrt{3}$

D $\frac{15}{4}$

E $\frac{15}{2}$

11. Box A contains one counter, and boxes B and C each contain two counters.

Three counters are removed, one at a time. Before each removal, one of the boxes that is not empty is chosen at random, with each nonempty box equally likely to be chosen. One counter is then removed from that box and is not replaced.

Given that the two counters remaining after the three removals are in the same box, what is the probability that the first counter removed came from box A?

A $\frac{1}{6}$

B $\frac{1}{3}$

C $\frac{3}{7}$

D $\frac{4}{7}$

E $\frac{3}{5}$

12. The points $A = (0, 0)$, $B = (4, 0)$ and $C = (2, t)$, where $t > 0$, are three vertices of a parallelogram. They are not necessarily consecutive vertices.

One diagonal of the parallelogram is twice as long as the other.

What is the sum of all possible distinct values of t^2?

A $\frac{71}{3}$

B 17

C $\frac{23}{3}$

D $\frac{68}{3}$

E $\frac{91}{3}$

F 38

13. A geometric progression $u_1, u_2, u_3, \ldots$ has a negative common ratio. It is given that

$$u_3 = 8, \qquad u_5 = 2.$$

What is $u_4 + u_6$?

A -20

B -5

C -4

D $-\frac{5}{2}$

E 5

14. The points A, B and C lie in that order on a horizontal straight line, with $AB = BC = 3\,\text{m}$.

A vertical pole of positive height has its base P on this line, distinct from A, B and C. The angles of elevation of the top of the pole from A, B and C are α, β and γ, respectively, and

$$5(\tan\alpha + \tan\gamma) = 12\tan\beta.$$

What is the sum of all the distinct possible values of BP, in metres?

A 2

B $3\sqrt{6} - \dfrac{5}{2}$

C $3\sqrt{6}$

D $2 + 3\sqrt{6}$

E $4 + 6\sqrt{6}$

15. For each real number k, consider the equation

$$\frac{3x^2 + 4x + 1}{x^2 + 1} = k.$$

What is the sum of all the values of k for which this equation has exactly one real solution for x?

A 3

B 4

C $2 - \sqrt{5}$

D $2 + \sqrt{5}$

E 7

16. A positive integer n has exactly six positive divisors, including 1 and n. The sum of these six divisors is 434.

What is the sum of all possible values of n?

A 187

B 244

C 325

D 569

E 598

17. The sequence with nth term $u_n = 2^n - 1$ can be generated by starting with 1 and repeatedly doubling the previous term and adding 1.

A student generates a list starting with 1, but uses each of the following incorrect rules exactly once, at two different steps:

- double the previous number and add 3;
- double the previous number and subtract 1.

At every other step, the student doubles the number just obtained and adds 1.

The tenth number in the student's list is 1047. What is the first incorrect number in the list?

A 31

B 63

C 65

D 131

E 129

18. The points $A = (1, 2)$ and $B = (4, 6)$ are adjacent vertices of a square.

The centre of the square has x-coordinate less than 1. What are the coordinates of the centre?

A $\left(\frac{5}{2}, 4\right)$

B $\left(\frac{1}{2}, \frac{11}{2}\right)$

C $\left(\frac{9}{2}, \frac{5}{2}\right)$

D $\left(-\frac{3}{2}, 7\right)$

E $\left(\frac{1}{2}, \frac{5}{2}\right)$

19. The function f is a quadratic polynomial. The tangent to $y = f(x)$ at the point where $x = 1$ has equation $y = 2x + 1$.

It is also given that

$$\int_0^2 f(x)\,dx = 10.$$

What is the value of $f'(2)$?

A -10

B 2

C 6

D 12

E 26

F 14

20. For each real number x, let $m(x)$ be the smaller of the two numbers

$$(x-1)^2+2 \qquad \text{and} \qquad 10-(x+1)^2.$$

What is the greatest possible value of $m(x)$?

A 2

B $6-2\sqrt{3}$

C 6

D $6+2\sqrt{3}$

E 10