1. The positive real numbers a and b satisfy $a : b = 3 : 2$.

What is the value of

$$\frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}} ?$$

A -5

B $-\frac{1}{5}$

C 0

D $\frac{1}{5}$

E 5

2. A sequence has at least three terms, all of which are positive real numbers. Consider the statement:

If the sequence is both an arithmetic progression and a geometric progression, then all its terms are equal.

Which of the following is the converse of this statement?

A If the terms are not all equal, then the sequence is neither an arithmetic progression nor a geometric progression.

B If the sequence is both an arithmetic progression and a geometric progression, then its terms are not all equal.

C If the sequence is not both an arithmetic progression and a geometric progression, then its terms are not all equal.

D If the terms are not all equal, then the sequence is not both an arithmetic progression and a geometric progression.

E If all the terms are equal, then the sequence is both an arithmetic progression and a geometric progression.

3. The angle θ satisfies

$$\cos\theta = -\sqrt{3}\sin\theta, \qquad 0^\circ < \theta < 180^\circ.$$

What is the value of θ?

A 30°

B 60°

C 120°

D 150°

E 330°

4. Consider the following claim about a positive integer n:

If n has an even number of positive divisors, then n is even.

Which of the following values of n is a counterexample to this claim?

A 1

B 2

C 3

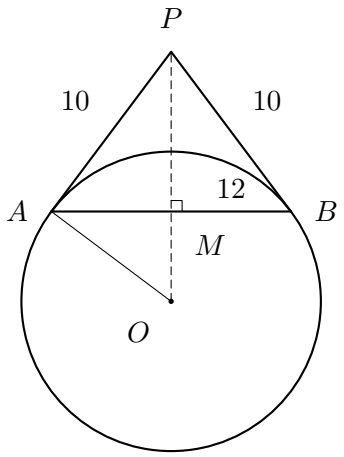
D 4

E 9

5. The lines PA and PB are tangents to a circle with centre O, touching it at A and B. The point M is the midpoint of the chord AB.

$$PA = PB = 10, \qquad AB = 12.$$

What is the radius of the circle?



A $\frac{15}{4}$
B 6
C $\frac{15}{2}$
D 8
E $\frac{40}{3}$
F 15

6. Two coins are tossed independently. Their probabilities of landing heads are p and q, where $0 < p < 1$ and $0 < q < 1$.

Consider the conditions:

P: $p + q = 1$.

Q: Given that the two coins show the same result, the probability that both show heads is $1/2$.

Which statement correctly describes P as a condition for Q?

A Necessary but not sufficient

B Both necessary and sufficient

C Neither necessary nor sufficient

D Sufficient but not necessary

7. The real number x satisfies

$$\log_2\left(\frac{x}{8}\right) = \sqrt{\log_2(8x)}.$$

What is the value of x?

A 2

B 6

C 64

D 128

E 512

8. Let a and b be nonzero real numbers such that $a+b$ and a^3+b^3 are rational. A student gives the following proof that ab is rational.

I. Write $s = a+b$ and $t = a^3+b^3$. Then s and t are rational.

II. Expanding s^3 gives $s^3 = t + 3abs$.

III. Since $a \neq 0$ and $b \neq 0$, we have $s \neq 0$.

IV. Hence $ab = (s^3 - t)/(3s)$ is rational.

Which of the following is correct?

A The first error is in line I

B The first error is in line II

C The first error is in line III

D The first error is in line IV

E All four lines I–IV are valid

9. A sequence is defined by

$$u_1 = 1, \qquad u_{n+1} = 2u_n + n \quad (n \geq 1).$$

It is given that $u_{11} = 3060$.

What is the value of $u_1 + u_2 + \cdots + u_{10}$?

A 1502

B 2993

C 3005

D 3004

E 3059

10. Let x and y be positive real numbers. Consider the statements

P: $x \geq 2$ and $y \geq 2$.

Q: $xy \geq x + y$.

Which of the following correctly describes P as a condition for Q?

A P is necessary but not sufficient for Q

B P is sufficient but not necessary for Q

C P is neither necessary nor sufficient for Q

D P is both necessary and sufficient for Q

11. Let a be a real number with $0 < a < 1$. The total area between the curve

$$y = \frac{x^2 - 1}{x^2},$$

the x-axis and the lines $x = a$ and $x = 1/a$ is 4.

What is a?

A There is no such value of a

B $2 + \sqrt{3}$

C 1

D $3 - 2\sqrt{2}$

E $2 - \sqrt{3}$

12. A circle has centre (h, k) and positive radius. Consider the statements

P: Both h and k are rational.

Q: The circle passes through at least three distinct points whose two coordinates are integers.

Which of the following correctly describes P as a condition for Q?

A P is necessary but not sufficient for Q

B P is both necessary and sufficient for Q

C P is neither necessary nor sufficient for Q

D P is sufficient but not necessary for Q

13. What is the total length of the parts of the range $0 \leq \theta < 2\pi$ for which both

$$2\sin^2\theta < 1 + \cos\theta \qquad \text{and} \qquad 2\cos^2\theta > 1 + \sin\theta$$

hold?

A $\frac{\pi}{3}$

B $\frac{4\pi}{3}$

C $\frac{\pi}{6}$

D $\frac{2\pi}{3}$

E $\frac{3\pi}{2}$

F $\frac{\pi}{2}$

14. A nonempty collection C consists of functions from the real numbers to the real numbers. Consider the statement:

> For every function f in C, there exist real numbers $x < y$ such that $f(x) \geq 0$ and $f(y) < 0$.

Which of the following is the logical negation of this statement?

A There is a function f in C such that, for every pair of real numbers $x < y$, both $f(x) < 0$ and $f(y) \geq 0$

B For every function f in C, there exist real numbers $x < y$ such that $f(x) < 0$ or $f(y) \geq 0$

C There exist real numbers $x < y$ such that, for every function f in C, $f(x) < 0$ or $f(y) \geq 0$

D There is a function f in C such that, for every pair of real numbers $x < y$, $f(x) < 0$ or $f(y) \geq 0$

E There is a function f in C such that, for every pair of real numbers $x < y$, $f(x) \leq 0$ or $f(y) > 0$

15. Two lamps, X and Y, are initially off. Each switches on once, at a time between 0 and 6 minutes. Their switching-on times x and y are chosen independently. The probability that (x, y) lies in any part of the square $0 \leq x \leq 6$, $0 \leq y \leq 6$ is proportional to that part's area.

Lamp X stays on for 2 minutes, and lamp Y stays on for 3 minutes. Each then switches off.

Given that there is a time when both lamps are on, what is the probability that X switches off before Y?

A $\frac{16}{47}$

B $\frac{20}{47}$

C $\frac{31}{72}$

D $\frac{31}{47}$

E $\frac{47}{72}$

F $\frac{1}{2}$

16. For a non-degenerate triangle, consider the statements

P: Its side lengths, written from shortest to longest, form an arithmetic progression.

Q: Its largest angle is $120°$.

Which statement correctly describes P as a condition for Q?

A P is necessary but not sufficient for Q

B P is neither necessary nor sufficient for Q

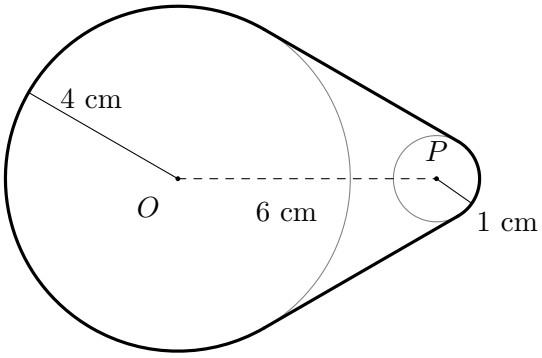
C P is both necessary and sufficient for Q

D P is sufficient but not necessary for Q

17. Two circular wheels have radii 4 cm and 1 cm, and their centres O and P are 6 cm apart.

A thin belt is pulled taut around the outside of both wheels, as shown. The belt does not cross itself. It consists of two straight common tangents and an arc of each wheel.

What is the total length of the belt, in centimetres?



A $5\pi + 6\sqrt{3}$

B $6\pi + 12$

C $6\pi + 3\sqrt{3}$

D $4\pi + 6\sqrt{3}$

E $6\pi + 6\sqrt{3}$

F $10\pi + 6\sqrt{3}$

18. A list contains 20 numbers, each of which is 0, 2, 6 or 9.

Exactly five of the numbers are 2, and exactly three are 9. The median of the list is 4.

What is the mean of the list?

A $\frac{73}{20}$

B $\frac{79}{15}$

C $\frac{79}{20}$

D 4

E $\frac{17}{4}$

19. Let $n \geq 3$, and let $a_1 < a_2 < \cdots < a_n$ be real numbers. Among all the positive differences $a_j - a_i$, where $i < j$, there are exactly $n - 1$ distinct values.

The following incomplete proof shows that $a_1, \ldots, a_n$ form an arithmetic progression.

Consider the list

$$a_n - a_1 > a_n - a_2 > \cdots > a_n - a_{n-1}.$$

[Missing argument]

This establishes

$$a_{n-1} - a_i = a_n - a_{i+1} \qquad (1 \leq i \leq n-2).$$

Rearranging shows that each of the first $n - 2$ consecutive gaps equals the last gap, $a_n - a_{n-1}$, proving the result.

Which argument correctly fills the gap without assuming the result to be proved?

A The first list contains $n - 1$ entries, and there are $n - 1$ consecutive gaps. Thus the consecutive gaps exhaust this list. Comparing them in order gives the required equalities.

B For $1 \leq i \leq n - 2$, we have $a_{n-1} - a_i < a_n - a_i$. Strict ordering alone then forces the left-hand value to be the next smaller entry of the first list.

C Subtract $a_n - a_{n-1}$ from every entry of the first list. Each result is a positive difference already in the list, so the list is unchanged. Matching entries gives the required equalities.

D All positive differences occur in the first list. The $n - 2$ distinct differences $a_{n-1} - a_i$ with $1 \leq i \leq n - 2$ are smaller than its largest entry, so they fill the other $n - 2$ entries. Match these two decreasing lists.

E Write $a_i = a_1 + (i-1)d$, where $d > 0$. The first list is $(n-1)d, (n-2)d, \ldots, d$, and the differences from a_{n-1} are $(n-2)d, \ldots, d$. Comparing these lists gives the required equalities.

20. For a real constant a, define $f(x) = x^3 + ax$ for all real x. It is known that

$$f(x+1) > f(x)$$

for every real x.

Consider the statements

I. $a > -\frac{1}{4}$.

II. For all real $u < v$, we have $f(u) < f(v)$.

III. Whenever u and v are distinct real numbers with $f(u) = f(v)$, we have $|u - v| < 1$.

Which statements must be true?

A I only

B II only

C III only

D I and II only

E I and III only

F II and III only

G I, II and III