1. A sequence is defined by

$$u_n = n\left(\frac{3}{4}\right)^{n-1} \qquad (n = 1, 2, 3, \ldots).$$

You may use the relation

$$\frac{u_{n+1}}{u_n} = \frac{3(n+1)}{4n}.$$

For which values of n does u_n take its greatest value?

A $n = 3$ only

B $n = 4$ only

C $n = 3$ and $n = 4$ only

D $n = 4$ and $n = 5$ only

E There is no greatest value

2. The functions f and g are given by

$$f(x) = \sqrt{x-1}, \qquad g(x) = \frac{1}{x-3},$$

where each function is defined wherever its formula has a real value.

Which of the following describes all real values of x for which $f(g(x))$ is defined?

A $x < 3$ or $x \geq 4$

B $3 < x \leq 4$

C $3 < x < 4$

D $3 \leq x \leq 4$

E $x \leq 4$, with $x \neq 3$

3. Let n be an integer satisfying $1 \leq n < 240$. The fraction $n/240$ can sometimes be written as a terminating decimal with exactly three digits after the decimal point, after all trailing zeros have been removed.

What is the sum of all values of n for which this is possible?

A 2280

B 2340

C 2400

D 4680

E 9480

4. Three faces of a cuboid meet at a vertex. The diagonals of these three faces have lengths $\sqrt{13}$, $\sqrt{20}$ and 5.

What is the volume of the cuboid?

A $\sqrt{29}$

B 24

C $48\sqrt{2}$

D $10\sqrt{65}$

E 576

5. What are all the real values of x satisfying

$$\int_1^x (t^2 - 4t + 3)\,\mathrm{d}t = 0\,?$$

A $x = 1$ only

B $x = 3$ only

C $x = 1$ or $x = 3$

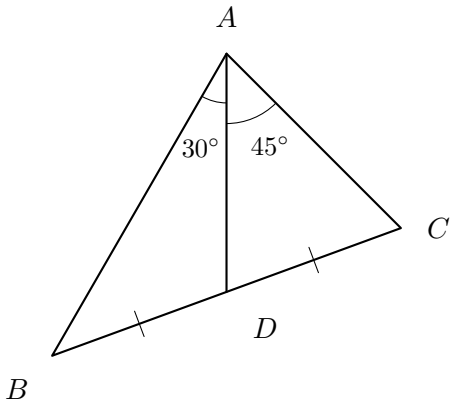
D $x = 0$ or $x = 3$

E $x = 1$ or $x = 4$

6. In triangle ABC, the point D is the midpoint of BC. It is given that

$$\angle BAD = 30^\circ, \qquad \angle DAC = 45^\circ.$$

What is the value of AB/AC?



A $\dfrac{\sqrt{2}}{2}$

B $\dfrac{\sqrt{6}}{3}$

C 1

D $\sqrt{2}$

E $\sqrt{3}$

7. For each nonzero real number x, let

$$u = x + \frac{1}{x}, \qquad v = x^2 + \frac{1}{x^2}.$$

You may use the identity $v = u^2 - 2$.

How many distinct real values of x satisfy

$$u^2 - 5u = v^2 - 5v\,?$$

A 1

B 3

C 4

D 5

E 6

8. The positive integer n is such that the sum of the coefficient of x and the coefficient of x^2 in the expansion of $(1 + x)^n$ is 45.

What is the coefficient of x^3 in this expansion?

A 9

B 36

C 84

D 120

E 504

9. Two solid blocks A and B have densities 8 g/cm^3 and 6 g/cm^3, respectively. Their masses are in the ratio $2:3$, and the sum of their volumes is 150 cm^3.

What is the mass of block A, in kilograms?

A 0.4 kg

B 0.48 kg

C 0.6 kg

D 1 kg

E 1.2 kg

10. For a real constant a, a curve has equation

$$y = \frac{x^2 + ax - a - 3}{\sqrt{x}}, \qquad x > 0.$$

For which values of a does the curve have both a local maximum and a local minimum?

A $a < -3$

B $-3 < a < 0$

C $-3 \leq a < 6 - 6\sqrt{2}$

D $-3 < a < 6 - 6\sqrt{2}$

E $-3 < a \leq 6 - 6\sqrt{2}$

F $a < 6 - 6\sqrt{2}$ or $a > 6 + 6\sqrt{2}$

11. The function f is defined for all real x by

$$f(x) = 4^x - 6 \cdot 2^x.$$

What is the real solution of

$$f(x+1) - f(x) = 9?$$

A $\frac{1}{2}\log_2 3$

B 1

C $\frac{1}{2}\log_2 5$

D $\log_2 3$

E 3

12. In triangle ABC, the points A and B have coordinates $(0, 0)$ and $(6, 0)$. The three lines drawn through the vertices perpendicular to their opposite sides meet at $H = (2, 3)$. Opposite sides may be extended when drawing these lines.

What is the area of triangle ABC?

A 4

B 8

C 9

D 16

E 18

13. The side lengths of a triangle are x, $x+1$ and $2x-1$. Its largest angle is $120°$.

What is the sum of all possible values of x?

A $\frac{7}{6}$

B $\frac{169}{30}$

C $\frac{35}{6}$

D 7

E $\frac{49}{6}$

14. A continuous function f satisfies $f(0) = f(1) = f(4) = 0$. Its graph lies above the x-axis for $0 < x < 1$ and below the x-axis for $1 < x < 4$.

The area between the graph and the x-axis is 5 for $0 \leq x \leq 1$ and 2 for $1 \leq x \leq 4$.

What is the value of

$$\int_0^4 (2f(x) - 3)\, dx?$$

A -18

B -6

C 2

D 3

E 6

15. Coins A and B have probabilities p and q of landing heads, respectively, where $0 < q < p < 1$. All tosses are independent.

It is given that $p + q = 13/12$. When each coin is tossed once, the probability that both land heads is $1/4$.

Coin A is now tossed twice and coin B once. Given that exactly two of these three tosses land heads, what is the probability that coin B lands tails?

A $\frac{1}{4}$

B $\frac{3}{8}$

C $\frac{2}{3}$

D $\frac{3}{4}$

E $\frac{6}{7}$

16. For $1 \leq x < 4$, the function f is defined by

$$f(x) = (x^2 - 4x)(x^2 - 4x + 6).$$

Which inequality describes the complete set of values of $y = f(x)$?

A $-9 \leq y \leq 0$

B $-9 < y < 0$

C $-8 \leq y < 0$

D $-9 \leq y < 0$

E $y \geq -9$

17. A circle has centre $(2, -3)$. The line $y = 2x + 1$ is tangent to the circle.

The other tangent parallel to this line meets the x-axis at P. What is the x-coordinate of P?

A $-\frac{15}{2}$

B $-\frac{1}{2}$

C $\frac{7}{2}$

D $\frac{9}{2}$

E $\frac{15}{2}$

18. Five real numbers, in increasing order, form an arithmetic progression. Their sum is 15, and the sum of their absolute values is 19.

What is the largest of the five numbers?

A $\frac{5}{2}$

B $\frac{11}{2}$

C 8

D $\frac{25}{3}$

E 10

19. Five real numbers x_1, x_2, x_3, x_4, x_5 have mean 3.

The ten products formed by choosing two different positions in this list have mean 8. For example, x_1x_2 is one of these products, and each pair of positions is used once.

What is the mean of $x_1^2, x_2^2, x_3^2, x_4^2, x_5^2$?

A 1

B 9

C 13

D 29

E 65

20. For a real number k, the equations

$$x^2 + kx - 2 = 0 \qquad \text{and} \qquad x^2 + x + k^2 - 3 = 0$$

have exactly one common real solution.

What is the sum of all possible values of k?

A -3

B $-\frac{3}{2}$

C $-\frac{1}{2}$

D 1

E $\frac{-3+\sqrt{17}}{4}$