1. Consider the following claim about a real number x:

$$\text{If } x > 1, \text{ then } 2^x > x^2.$$

Which of the following values of x is a counterexample to this claim?

A -2

B 6

C 1

D 2

E 5

2. The positive real numbers a and b satisfy

$$a^2 + ab = 12, \qquad b^2 + ab = 18.$$

What is the value of $a + b$?

A $\sqrt{6}$
B $\sqrt{15}$
C $\sqrt{30}$
D $\sqrt{12} + \sqrt{18}$
E 30

3. For which values of the real constant k do the graphs

$$y = \frac{x^2 - 4}{x - 2} \quad (x \neq 2) \qquad \text{and} \qquad y = kx$$

have no point in common?

A $k = 1$ only

B $k = 1$ or $k = 2$

C $k = 2$ only

D $k = -1$ or $k = 1$

E $k = 0$ or $k = 2$

4. A data set consists of five distinct real numbers. Consider the statements

P: The mean equals the median.

Q: The sum of the smallest and largest numbers equals twice the median.

Which of the following is correct?

A P is both necessary and sufficient for Q.

B P is neither necessary nor sufficient for Q.

C P is necessary but not sufficient for Q.

D P is sufficient but not necessary for Q.

5. A rectangle has width w, height h and diagonal length d. Consider the statement

$$\text{If } d > 10, \text{ then } w > 6 \text{ or } h > 8.$$

Which of the following is its contrapositive?

A If $w > 6$ or $h > 8$, then $d > 10$.

B If $d \leq 10$, then $w \leq 6$ and $h \leq 8$.

C If $w \leq 6$ or $h \leq 8$, then $d \leq 10$.

D If $w < 6$ and $h < 8$, then $d < 10$.

E If $w \leq 6$ and $h \leq 8$, then $d \leq 10$.

6. The function f is defined for all real x by

$$f(x) = ab^x + c,$$

where a and c are real constants and $b > 0$. It is given that

$$f(0) = 5, \qquad f(1) = 7, \qquad f(3) = 19.$$

What is the value of $f(2)$?

A 1

B 9

C 11

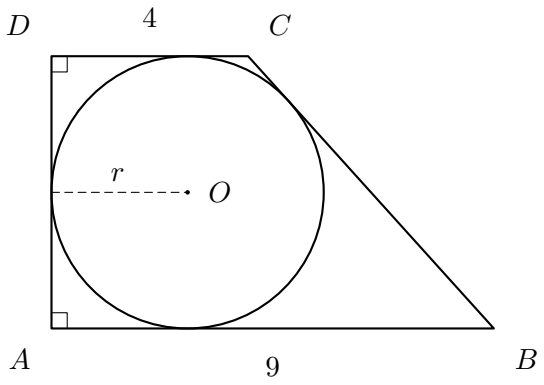
D 13

E 14

7. In trapezium $ABCD$, the sides AB and DC are parallel, and AD is perpendicular to them. Also, $AB = 9$ and $DC = 4$. A circle lies inside the trapezium and is tangent to all four sides, as shown.

What is the radius of the circle?

You may use the fact that the two tangent segments from an external point to a circle have equal lengths.



A $\frac{18}{13}$

B $\frac{5}{2}$

C $\frac{36}{13}$

D $\frac{13}{4}$

E $\frac{72}{13}$

8. A non-empty collection of circles is drawn in a plane. Consider the statement:

For every straight line through the origin, there is at least one circle in the collection that meets the line at two distinct points.

Which of the following is the negation of this statement?

A There is a straight line through the origin that has no point in common with any circle in the collection

B For every straight line through the origin, there is a circle in the collection that meets the line at at most one point

C There is a straight line through the origin and a circle in the collection that meet at at most one point

D There is a straight line through the origin that meets every circle in the collection at at most one point

E For every circle in the collection, there is a straight line through the origin that meets it at at most one point

9. For $1 \leq x \leq 2$, the curve C has equation

$$y = \frac{x^3 + 1}{x^2}.$$

A horizontal line $y = k$ has the following property: within the strip $1 \leq x \leq 2$, the total area between C and the line where C is above the line equals the total area between them where C is below the line.

What is k?

A $\frac{3}{2}$

B $\frac{7}{4}$

C 2

D $\frac{17}{8}$

E $\frac{9}{4}$

10. Let a and b be positive integers. Consider the statements

P: The highest common factor of a and b is 1.

Q: $a^2 + ab + b^2$ is prime.

Which of the following is correct?

A P is sufficient but not necessary for Q

B P is necessary but not sufficient for Q

C P is both necessary and sufficient for Q

D P is neither necessary nor sufficient for Q

11. All the positive integers whose usual base-ten representations contain only the digits 0 and 1 are listed in increasing order:

$$1,\ 10,\ 11,\ 100,\ 101,\ 110,\ 111, \ldots .$$

What is the sum of the first 31 integers in this list?

A 88888

B 166665

C 177760

D 177776

E 355552

12. Bag A contains three red balls and two blue balls. Bag B contains two red balls and one blue ball.

One ball is chosen at random from bag A and transferred to bag B. Two balls are then chosen at random from bag B, without replacement. At each choice, all balls then available are equally likely to be chosen.

Given that the two balls chosen from bag B have the same colour, what is the probability that the transferred ball is one of those two balls?

A $\frac{4}{15}$

B $\frac{1}{2}$

C $\frac{7}{12}$

D $\frac{3}{5}$

E $\frac{8}{13}$

F $\frac{9}{13}$

13. Let a and b be real numbers satisfying $a + b = 2$. A student gives the following proof that $a^4 + b^4 \geq 2$.

I. Since $(a - b)^2 \geq 0$, the identity $4ab = (a + b)^2 - (a - b)^2$ gives $ab \leq 1$.

II. Hence $a^2 + b^2 = (a + b)^2 - 2ab \geq 2$.

III. Squaring the inequality in line II gives $(a^2 + b^2)^2 \geq 4$.

IV. Squaring $ab \leq 1$ gives $a^2b^2 \leq 1$.

V. Therefore $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 \geq 4 - 2 = 2$.

Which of the following is correct?

A The first error is in line I

B The first error is in line II

C The first error is in line III

D The first error is in line IV

E The first error is in line V

F All five lines I–V are valid

14. The quadratic equation

$$x^2 + ax + b = 0$$

has two distinct real roots, where a and b are real constants. Consider the statements

P: $|a| + |b| < 1$.

Q: Both roots satisfy $-1 < x < 1$.

Which of the following is correct?

A P is sufficient but not necessary for Q

B P is neither necessary nor sufficient for Q

C P is necessary but not sufficient for Q

D P is both necessary and sufficient for Q

15. Find the sum of all the distinct solutions θ of

$$\sqrt{1+\sin\theta}+\sqrt{1-\sin\theta}=\sqrt{2}\,(\sin\theta+\cos\theta)$$

for which $0 \leq \theta < 2\pi$.

A $\dfrac{\pi}{6}$

B $\dfrac{\pi}{2}$

C $\dfrac{2\pi}{3}$

D $\dfrac{4\pi}{3}$

E $\dfrac{13\pi}{6}$

F $\dfrac{10\pi}{3}$

16. Let x and y be positive real numbers. Consider the conditions

$$P: \quad x = y,$$

$$Q: \quad \log_2(x+y) = 1 + \frac{\log_2 x + \log_2 y}{2}.$$

Which statement correctly describes P as a condition for Q?

A Sufficient but not necessary

B Neither necessary nor sufficient

C Necessary but not sufficient

D Both necessary and sufficient

17. The $2n$ distinct positive real numbers $a_1 < a_2 < \cdots < a_{2n}$ are divided into n pairs. The sum of the numbers in each pair is found, and these n sums are multiplied.

Consider this proof that the greatest product is obtained by pairing a_1 with a_{2n}, a_2 with a_{2n-1}, and so on.

Choose a pairing with greatest product. Write $a = a_1$ and $d = a_{2n}$. Suppose that a and d are not paired together: their pairs are (a, b) and (c, d). Then $a < b < d$ and $a < c < d$.

[Missing argument]

This contradiction forces a and d to be paired together. The same argument applies to the remaining numbers, proving the result.

Which option correctly supplies the missing argument?

A Replace the two pairs by (a, c) and (b, d). Their sum-product increases by $(c - b)(d - a)$, which is always positive.

B Replace the two pairs by (a, d) and (b, c). Their sum-product increases by $(d - b)(c - a) > 0$, while every other positive factor is unchanged.

C Replace the two pairs by (a, d) and (b, c). Their sum-product increases by $(d - c)(b - a) > 0$, while every other positive factor is unchanged.

D Replace the two pairs by (a, d) and (b, c). Both new pair sums are greater than both original pair sums, so the product increases.

E Replace the two pairs by (a, c) and (b, d). The total of the pair sums is unchanged, so their product increases.

18. A non-empty list of real numbers is used to form two new lists.

In the first new list, each number x is replaced by $(x-2)^2$. In the second new list, each original number x is replaced by $(x-6)^2$.

The means of the two new lists are 13 and 5 respectively.

What is the mean of the original list?

A 3

B 4

C 5

D 6

E 8

F 9

19. The positive integers a and b have highest common factor 1. Both ab and $a + b$ are perfect squares.

Which of the following statements must be true?

I a and b are both perfect squares.

II $a + b$ is odd.

III At least one of a, b is divisible by 5.

A None of them

B I only

C II only

D III only

E I and III only

F I and II only

G II and III only

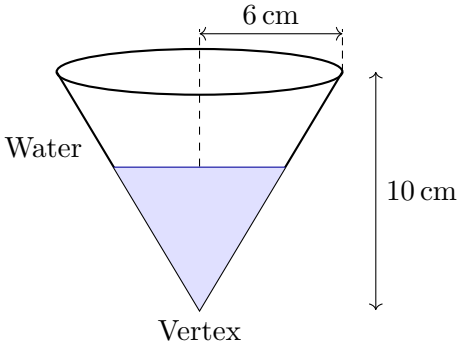
H I, II and III

20. A tank is a right circular cone with its vertex at the bottom. Its vertical height is 10 cm and its top radius is 6 cm. Initially it contains some water.

Water is added at a constant volume per minute. In one time interval, the depth of the water increases by 2 cm. In the next time interval of the same duration, it increases by 1 cm. The tank does not overflow.

What is the depth of the water, in centimetres, at the end of the first of these two intervals?

The volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.



A $\dfrac{15+\sqrt{141}}{6}$

B $\dfrac{15-\sqrt{141}}{6}$

C $\dfrac{3+\sqrt{141}}{6}$

D $\dfrac{21+\sqrt{141}}{6}$

E $\dfrac{5}{2}$

F 5