1. A map has scale $1 : 25\,000$. It is enlarged so that every length on the new map is 20% greater than the corresponding length on the original map.

A route measures 7.2 cm on the enlarged map. A walker follows this route at a constant speed of $4\,\text{km}\,\text{h}^{-1}$.

How many minutes does the walk take?

A 18

B 21.6

C 22.5

D 27

E 32.4

2. The function f is defined by

$$f(x) = x^2 + ax + b,$$

where a and b are real constants. It satisfies

$$f(0) = f(4) \qquad \text{and} \qquad f(f(1)) = 1.$$

What is the sum of all possible values of b?

A 4

B 5

C 7

D 9

E 20

F 25

3. A point P lies on the curve

$$y = \sqrt{x}, \qquad x > 0.$$

The normal to the curve at P meets the x-axis at $\left(\frac{9}{2}, 0\right)$.

What is the y-coordinate of P?

A $\frac{3}{2}$

B 2

C $\frac{3}{\sqrt{2}}$

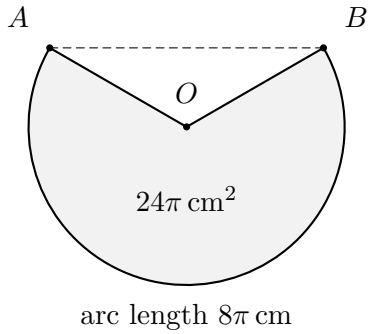
D $\frac{9}{4}$

E 4

F $\frac{9}{2}$

4. A circular sector with centre O has area $24\pi\,\text{cm}^2$. The length of its curved arc from A to B is $8\pi\,\text{cm}$.

What is the length of the straight line segment AB?



A $3\,\text{cm}$

B $3\sqrt{3}\,\text{cm}$

C $6\,\text{cm}$

D $6\sqrt{2}\,\text{cm}$

E $6\sqrt{3}\,\text{cm}$

F $12\,\text{cm}$

5. The points A and B have coordinates $(0, 0)$ and $(5, 0)$, respectively.

There are two points on the line $y = x/2$ whose distance from A is twice their distance from B.

What is the distance between these two points?

A $\frac{4}{3}$

B $\frac{8}{3}$

C $\frac{4\sqrt{5}}{3}$

D $\frac{8\sqrt{5}}{3}$

E $\frac{10\sqrt{5}}{3}$

6. A sequence is defined by $u_1 = 2$ and

$$u_{n+1} = \sqrt{u_n^2 + 5} \qquad (n \geq 1).$$

What is the least value of n for which $u_n > 8$?

A 12

B 13

C 14

D 15

E 3

7. The function f is defined by

$$f(x) = ax^2 + bx + c,$$

where a, b and c are real constants; a is allowed to be zero. For every real number x,

$$f(x^2) = f(x)^2 - 2f(x) + 2.$$

How many ordered triples (a, b, c) satisfy this condition?

A 1

B 2

C 3

D 4

E 6

F 8

8. A coin has a fixed probability of landing heads, and successive tosses are independent. The probability that two tosses give different outcomes is 3/8.

What is the probability that three tosses all give the same outcome?

A $\frac{3}{16}$

B $\frac{1}{4}$

C $\frac{7}{16}$

D $\frac{9}{16}$

E $\frac{5}{8}$

F $\frac{13}{16}$

9. When a polynomial $P(x)$ is divided by $x^2 - x - 1$, the remainder is $3x + 2$.

What is the remainder when $(x - 1)P(x)$ is divided by $x^2 - x - 1$?

A $-x - 2$

B $2x - 5$

C $2x + 1$

D $3x + 2$

E $3x^2 - x - 2$

10. Triangle ABC has $AB = 6$ and area 18. The points M and N are the midpoints of BC and CA, respectively, and AM is perpendicular to BN.

What is the value of $|AC^2 - BC^2|$?

A $12\sqrt{5}$

B $18\sqrt{5}$

C 36

D $36\sqrt{5}$

E $72\sqrt{5}$

F 180

11. The function F is defined by

$$F(x) = \int_0^x (t^2 - 2t)\,\mathrm{d}t, \qquad -1 \le x \le 4.$$

Which of the following describes the range of F?

A $-\dfrac{4}{3} \le F(x) \le 0$

B $-\dfrac{4}{3} \le F(x) \le \dfrac{16}{3}$

C $0 \le F(x) \le \dfrac{16}{3}$

D $-\dfrac{4}{3} \le F(x) \le 4$

E $-\dfrac{16}{3} \le F(x) \le \dfrac{4}{3}$

12. A sequence is defined by

$$u_1 = 3, \qquad u_{n+1} = u_n(u_n + 1) \quad (n \geq 1).$$

Let $M = u_{10}$. Which of the following is equal to

$$\frac{1}{u_1 + 1} + \frac{1}{u_2 + 1} + \cdots + \frac{1}{u_9 + 1}\,?$$

A $\dfrac{1}{M}$

B $\dfrac{1}{3} - \dfrac{1}{M+1}$

C $1 - \dfrac{3}{M}$

D $\dfrac{1}{3} - \dfrac{1}{M}$

E $\dfrac{1}{3} - \dfrac{1}{M(M+1)}$

F $\dfrac{1}{3}$

13. A list of eight numbers has mean 10 and range 12.

The smallest number is accidentally recorded a second time. The resulting list of nine numbers has mean 9.

What is the largest number in the original list?

A 1

B 12

C 13

D 20

E 22

14. The angle θ satisfies $\pi/2 < \theta < \pi$ and

$$3\sin^2\theta + 4\sin\theta\cos\theta = 4\cos^2\theta.$$

What is the value of

$$\frac{1}{\sin\theta\cos\theta}\,?$$

A $-\frac{5}{2}$

B -2

C $-\frac{2}{5}$

D $\frac{13}{6}$

E $\frac{5}{2}$

15. For which values of the real constant a does

$$x^2 + \frac{1}{x^2} - 2a\left(x + \frac{1}{x}\right) + 6a - 3 \geq 0$$

hold for every positive real number x?

A $a \geq \frac{1}{2}$

B $1 \leq a \leq 5$

C $\frac{1}{2} \leq a \leq 2$

D $\frac{1}{2} \leq a \leq 5$

E $0 \leq a \leq 5$

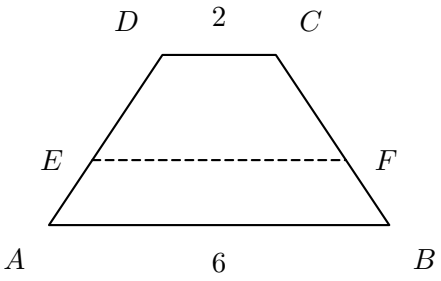
F $\frac{1}{2} < a < 5$

16. In trapezium $ABCD$, the sides AB and DC are parallel, with

$$AB = 6, \qquad DC = 2.$$

The points E and F lie on AD and BC, respectively. The segment EF is parallel to AB and divides the trapezium into two regions of equal area.

What is the length of EF?



A $2\sqrt{3}$

B 4

C $2\sqrt{5}$

D $3\sqrt{2}$

E 5

F $\sqrt{10}$

17. For each real number a, the curve C_a has equation

$$y = \frac{1}{4}x^4 - \frac{3}{2}x^2 + ax, \qquad 0 \leq x \leq 3.$$

Let $M(a)$ be the greatest absolute value of the gradient of C_a on this range of x.

For which value of a is $M(a)$ as small as possible?

A -18

B -10

C -9

D -8

E 0

F 8

18. The polynomial P is defined by

$$P(x) = (x + 2)^5.$$

What is the coefficient of x^2 in $(1 - x)P(2x - 1)$?

A 5

B 10

C 30

D 40

E 50

F 160

19. A positive divisor d of 360 is chosen at random, with every positive divisor equally likely. What is the probability that the highest common factor of d and $360/d$ is 6?

A $\frac{1}{12}$

B $\frac{1}{6}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

F $\frac{2}{3}$

20. For a real constant a, consider the equation

$$|x^2 - a| = |x - 2|.$$

What is the sum of all values of a for which this equation has exactly three distinct real solutions?

A $-\frac{1}{2}$

B $\frac{7}{4}$

C $\frac{7}{2}$

D 4

E $\frac{23}{4}$