1. C

The route's length on the original map is

$$\frac{7.2}{1.2} = 6\,\text{cm}.$$

Hence its actual length is $6 \times 25\,000 = 150\,000\,\text{cm} = 1.5\,\text{km}$. The walking time is therefore

$$\frac{1.5}{4} \times 60 = \boxed{22.5 \text{ minutes}}.$$

2. D

The first condition gives $b = 16 + 4a + b$, so $a = -4$ and $f(1) = b - 3$. Thus the second condition is

$$(b-3)^2 - 4(b-3) + b = 1.$$

Rearranging gives

$$b^2 - 9b + 20 = (b-4)(b-5) = 0.$$

Both possibilities work: if $b = 4$, then $f(1) = 1$; if $b = 5$, then $f(1) = 2$ and $f(2) = 1$. Therefore the required sum is $4 + 5 = \boxed{9}$.

3. B

Write $P = (p, \sqrt{p})$, where $p > 0$. The tangent gradient is $1/(2\sqrt{p})$, so the normal gradient is $-2\sqrt{p}$. Using the two points on the normal gives

$$0 - \sqrt{p} = -2\sqrt{p}\left(\frac{9}{2} - p\right).$$

Since $\sqrt{p} > 0$, division gives $1 = 9 - 2p$, so $p = 4$. The requested coordinate is $\sqrt{4} = \boxed{2}$.

Indeed the normal at $(4, 2)$ is $y - 2 = -4(x - 4)$, whose x-intercept is $9/2$.

4. E

Let the radius be r centimetres and the sector angle be θ radians. Then

$$r\theta = 8\pi, \qquad \frac{1}{2}r^2\theta = 24\pi.$$

Dividing the second equation by the first gives $r/2 = 3$, so $r = 6$ and $\theta = 4\pi/3$.

The smaller angle AOB is therefore $2\pi - 4\pi/3 = 2\pi/3$. By the cosine rule,

$$AB^2 = 6^2 + 6^2 - 2 \cdot 6 \cdot 6 \cos\frac{2\pi}{3} = 72 + 36 = 108.$$

Hence $AB = \boxed{6\sqrt{3}\,\text{cm}}$.

5. C

A point on the line has coordinates $(t, t/2)$. Squaring the distance condition gives

$$t^2 + \frac{t^2}{4} = 4\left((t-5)^2 + \frac{t^2}{4}\right).$$

Multiplying by 4 and simplifying yields

$$15t^2 - 160t + 400 = 0, \qquad (3t-20)(t-4) = 0.$$

Thus the two points are $(4, 2)$ and $(20/3, 10/3)$. Their distance is

$$\sqrt{\left(\frac{8}{3}\right)^2 + \left(\frac{4}{3}\right)^2} = \boxed{\frac{4\sqrt{5}}{3}}.$$

Both points satisfy the original condition: the squared distances from A and B are respectively $20, 5$ and $500/9, 125/9$. All distances are nonnegative, so squaring has introduced no additional points.

6. C

Squaring the recurrence gives $u_{n+1}^2 = u_n^2 + 5$. Hence the squared terms form an arithmetic progression, with

$$u_n^2 = 4 + 5(n-1) = 5n - 1.$$

All terms are positive, so $u_n > 8$ is equivalent to

$$5n - 1 > 64, \qquad n > 13.$$

The least such integer is $\boxed{14}$. In particular, $u_{13} = 8$ does not meet the strict inequality, while $u_{14} = \sqrt{69} > 8$.

7. D

Put $g(x) = f(x) - 1 = ax^2 + bx + d$, where $d = c - 1$. The identity becomes

$$g(x^2) = g(x)^2.$$

Comparing coefficients in

$$ax^4 + bx^2 + d = a^2x^4 + 2abx^3 + (b^2 + 2ad)x^2 + 2bdx + d^2$$

gives

$$a = a^2, \quad ab = 0, \quad b = b^2 + 2ad, \quad bd = 0, \quad d = d^2.$$

There are three short cases.

If $a \neq 0$, then $a = 1$, $b = 0$ and $d = 0$, so $g(x) = x^2$.

If $a = 0$ and $b \neq 0$, then $b = b^2$ gives $b = 1$, while $bd = 0$ gives $d = 0$. Thus $g(x) = x$.

If $a = b = 0$, then $d = d^2$, so $g(x) = 0$ or $g(x) = 1$.

Consequently the possible functions are

$$f(x) = x^2 + 1, \quad x + 1, \quad 1, \quad 2.$$

Each satisfies the identity directly, and their coefficient triples are distinct. The answer is therefore $\boxed{4}$.

8. C

Let p be the probability of heads and let $q = 1 - p$. The given probability is

$$pq + qp = \frac{3}{8}, \qquad pq = \frac{3}{16}.$$

In three tosses, the probability of a mixture of heads and tails is

$$3p^2q + 3pq^2 = 3pq(p + q) = 3pq = \frac{9}{16}.$$

The requested probability is therefore

$$1 - \frac{9}{16} = \boxed{\frac{7}{16}}.$$

Equivalently, it is $p^3 + q^3 = (p + q)^3 - 3pq(p + q)$. The data are possible: $p = 1/4$ or $p = 3/4$ gives the stated two-toss probability.

9. C

For some polynomial $Q(x)$,

$$P(x) = (x^2 - x - 1)Q(x) + 3x + 2.$$

Multiplying by $x - 1$, the first term remains divisible by $x^2 - x - 1$. For the remaining term,

$$(x - 1)(3x + 2) = 3x^2 - x - 2 = 3(x^2 - x - 1) + 2x + 1.$$

Since $2x + 1$ has degree less than 2, the required remainder is $\boxed{2x + 1}$.

10. D

Choose coordinates $A = (-3, 0)$ and $B = (3, 0)$, with $C = (x, h)$ above the x-axis. The area gives

$$\frac{1}{2} \cdot 6 \cdot h = 18, \qquad h = 6.$$

The midpoints are

$$M = \left(\frac{x+3}{2}, 3\right), \qquad N = \left(\frac{x-3}{2}, 3\right).$$

Neither median can be vertical in a perpendicular pair here: the other would have to be horizontal, but both join height 0 to height 3. Their gradients are therefore

$$\frac{6}{x+9} \quad \text{and} \quad \frac{6}{x-9}.$$

Perpendicularity gives

$$\frac{36}{x^2 - 81} = -1, \qquad x^2 = 45, \qquad |x| = 3\sqrt{5}.$$

Finally,

$$AC^2 - BC^2 = ((x+3)^2 + 36) - ((x-3)^2 + 36) = 12x.$$

Thus the required value is $\boxed{36\sqrt{5}}$. Both choices $x = \pm 3\sqrt{5}$ produce valid nondegenerate triangles and the same absolute difference.

11. B

By the Fundamental Theorem of Calculus,

$$F'(x) = x^2 - 2x = x(x-2).$$

Thus F is increasing for $-1 < x < 0$, decreasing for $0 < x < 2$, and increasing for $2 < x < 4$. Its extreme values on the given domain occur among $x = -1, 0, 2, 4$.

Integration gives $F(x) = x^3/3 - x^2$, so

$$F(-1) = -\frac{4}{3}, \qquad F(0) = 0, \qquad F(2) = -\frac{4}{3}, \qquad F(4) = \frac{16}{3}.$$

The minimum is $-4/3$ and the maximum is $16/3$. Both are attained, and the polynomial takes every value between them. Hence its range is

$$\boxed{-\frac{4}{3} \le F(x) \le \frac{16}{3}}.$$

12. D

Every term is positive, so the following divisions are valid. The recurrence gives

$$\frac{1}{u_n} - \frac{1}{u_{n+1}} = \frac{1}{u_n} - \frac{1}{u_n(u_n+1)} = \frac{1}{u_n+1}.$$

Consequently the sum telescopes:

$$\begin{aligned}\sum_{n=1}^{9} \frac{1}{u_n+1} &= \left(\frac{1}{u_1} - \frac{1}{u_2}\right) + \left(\frac{1}{u_2} - \frac{1}{u_3}\right) + \cdots + \left(\frac{1}{u_9} - \frac{1}{u_{10}}\right) \\ &= \frac{1}{u_1} - \frac{1}{u_{10}} = \boxed{\frac{1}{3} - \frac{1}{M}}.\end{aligned}$$

There is no need to calculate the large value of M.

13. C

The original sum is $8 \cdot 10 = 80$, and the sum after the extra entry is $9 \cdot 9 = 81$. Thus the smallest number is

$$81 - 80 = 1.$$

Since the range is 12, the largest number is $1 + 12 = \boxed{13}$.

The conditions are consistent. For example, the list $1, 11, 11, 11, 11, 11, 11, 13$ has sum 80 and range 12, and recording its smallest number again gives sum 81 across nine entries.

14. A

The specified quadrant gives $\cos\theta < 0$, so division by $\cos^2\theta$ is valid. Put $t = \tan\theta$. Then

$$3t^2 + 4t - 4 = 0, \qquad (3t-2)(t+2) = 0.$$

In the second quadrant $t < 0$, so $t = -2$, rather than $2/3$.

Using $\sin^2\theta + \cos^2\theta = 1$,

$$\frac{1}{\sin\theta\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \tan\theta + \frac{1}{\tan\theta} = -2 - \frac{1}{2} = \boxed{-\frac{5}{2}}.$$

There is exactly one second-quadrant angle with tangent -2, and neither sine nor cosine is zero there.

15. D

Put $t = x + 1/x$. Since

$$t - 2 = \frac{(x-1)^2}{x} \geq 0,$$

we have $t \geq 2$. Conversely, every $t \geq 2$ occurs: the roots of $x^2 - tx + 1 = 0$ are positive (including the repeated root $x = 1$ when $t = 2$).

Using $x^2 + x^{-2} = t^2 - 2$, the required condition is

$$q(t) = t^2 - 2at + 6a - 5 \geq 0 \qquad \text{for every } t \geq 2.$$

If $a \leq 2$, the vertex $t = a$ lies at or to the left of the allowed range. The minimum is therefore $q(2) = 2a - 1$, giving $1/2 \leq a \leq 2$.

If $a > 2$, the vertex is allowed and the minimum is

$$q(a) = -a^2 + 6a - 5 = (a-1)(5-a).$$

In this case nonnegativity gives $2 < a \leq 5$.

Combining the two cases gives $\boxed{\frac{1}{2} \leq a \leq 5}$. Both endpoints are included because the original inequality is non-strict.

16. C

Extend AD and BC to meet at T on the side of DC away from AB. The triangles with vertex T and bases DC, EF and AB are similar.

Write $EF = w$. Their areas are proportional to 2^2, w^2 and 6^2. Hence for some $k > 0$, the areas of the upper and lower parts of the trapezium are

$$k(w^2 - 4) \qquad \text{and} \qquad k(36 - w^2).$$

They are equal, so

$$w^2 - 4 = 36 - w^2, \qquad w^2 = 20.$$

Since w is a length, $\boxed{w = 2\sqrt{5}}$. This lies strictly between 2 and 6, so the corresponding parallel segment lies inside the trapezium.

17. D

The gradient is

$$g(x) = x^3 - 3x + a.$$

To find its greatest and least values, differentiate again:

$$g'(x) = 3x^2 - 3.$$

Thus g decreases for $0 < x < 1$ and increases for $1 < x < 3$. Its values at the endpoints and the one stationary input are

$$g(0) = a, \qquad g(1) = a - 2, \qquad g(3) = a + 18.$$

Consequently the gradient ranges from $a - 2$ to $a + 18$, and

$$M(a) = \max\{|a - 2|, |a + 18|\}.$$

These two extreme gradients differ by 20. If both had absolute value less than 10, their difference would be less than 20. Hence $M(a) \geq 10$.

For $a = -8$, the extreme gradients are -10 and 10, so $M(-8) = 10$. Equality is possible only when the two extreme gradients are -10 and 10. Thus the required value is $\boxed{-8}$.

18. C

First substitute $2x - 1$ for the whole input of P:

$$P(2x-1) = ((2x-1)+2)^5 = (1+2x)^5.$$

The coefficients of x and x^2 in this expansion are, respectively,

$$\binom{5}{1}2 = 10, \qquad \binom{5}{2}2^2 = 40.$$

To obtain x^2 after multiplying by $1 - x$, there are two contributions: the x^2 term multiplied by 1, and the x term multiplied by $-x$.

The required coefficient is therefore

$$40 - 10 = \boxed{30}.$$

19. B

Since $360 = 2^3 3^2 5$, each divisor has a unique form

$$d = 2^i 3^j 5^k, \qquad 0 \le i \le 3, \quad 0 \le j \le 2, \quad 0 \le k \le 1,$$

where the exponents are integers. There are $4 \cdot 3 \cdot 2 = 24$ equally likely divisors.

The complementary divisor is $2^{3-i} 3^{2-j} 5^{1-k}$. For their highest common factor to be $6 = 2 \cdot 3$:

- the smaller of i and $3 - i$ must be 1, so $i = 1$ or $i = 2$;
- the smaller of j and $2 - j$ must be 1, so $j = 1$;
- the smaller of k and $1 - k$ is automatically 0 for either allowed k.

There are therefore $2 \cdot 1 \cdot 2 = 4$ favourable divisors. The probability is $\boxed{4/24 = 1/6}$.

20. E

For real numbers, equal absolute values mean that the numbers are equal or opposite. Thus the solution set is the union of the real roots of

$$q_1(x) = x^2 - x + 2 - a = 0, \qquad q_2(x) = x^2 + x - 2 - a = 0.$$

Their discriminants are $4a - 7$ and $4a + 9$, respectively.

If $a < 7/4$, the first quadratic has no real roots, so there are at most two distinct solutions altogether.

If $a = 7/4$, the first quadratic has one repeated root, $x = 1/2$, while the second has two distinct real roots. Also $q_2(1/2) = -3 \neq 0$, so there are exactly three distinct solutions.

If $a > 7/4$, each quadratic has two distinct real roots. There are exactly three roots in their union precisely when they have one common root. Any common root satisfies

$$q_2(x) - q_1(x) = 2x - 4 = 0,$$

so $x = 2$. Substitution into either quadratic gives $a = 4$. At this value the quadratics factor as

$$q_1(x) = (x - 2)(x + 1), \qquad q_2(x) = (x - 2)(x + 3),$$

so their union is indeed the three-element set with values $-3, -1, 2$.

The only possible parameters are $7/4$ and 4, whose sum is

$$\boxed{\frac{23}{4}}.$$