1. For a real number $x \neq 2$, what is the value of

$$\frac{x^3 - 8}{x - 2} - x(x + 2)\,?$$

A -4

B 0

C 2

D 4

E 8

2. A real sequence $u_1, u_2, u_3, \ldots$ has the following property:

For every positive integer n, if $u_n > 0$, then $u_{n+1} > 0$.

Which of the following is an equivalent way of stating this property? Each option applies to every positive integer n.

A If $u_{n+1} > 0$, then $u_n > 0$.

B If $u_n \leq 0$, then $u_{n+1} \leq 0$.

C If $u_{n+1} \leq 0$, then $u_n \leq 0$.

D If $u_{n+1} < 0$, then $u_n < 0$.

E If $u_n > 0$, then $u_{n+1} \geq 0$.

3. An angle θ satisfies

$$\frac{\cos\theta}{1-\sin\theta} = 2.$$

What is the value of $\sin\theta$?

A $-\frac{3}{5}$

B $\frac{1}{5}$

C $\frac{3}{5}$

D $\frac{4}{5}$

E 1

4. A triangle has side lengths a, b and c, where $a \leq b \leq c$. Consider the statements

P: $a + b > \sqrt{2}\, c$.

Q: The angle opposite the side of length c is acute.

Which of the following is correct?

A P is necessary but not sufficient for Q.

B P is sufficient but not necessary for Q.

C P is both necessary and sufficient for Q.

D P is neither necessary nor sufficient for Q.

5. Consider the following claim about positive integers m and n:

If $m + n$ is prime, then $mn + 1$ is prime.

Which of the following is a counterexample to this claim?

A $m = 2, \quad n = 3$

B $m = 4, \quad n = 8$

C $m = 1, \quad n = 6$

D $m = 3, \quad n = 8$

E $m = 2, \quad n = 7$

6. A bag contains four balls: two red, one blue and one green. Three balls are drawn one after another without replacement, with each remaining ball equally likely to be drawn.

Given that the first and third balls drawn have different colours, what is the probability that the second ball is red?

A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{2}{3}$

F $\frac{4}{5}$

7. A sequence is defined by

$$u_1 = 1, \qquad u_{n+1} = 3u_n + 2 \quad (n \geq 1).$$

What is the value of

$$\frac{1}{u_1 + 1} + \frac{1}{u_2 + 1} + \frac{1}{u_3 + 1} + \frac{1}{u_4 + 1} + \frac{1}{u_5 + 1}\,?$$

A $\dfrac{121}{486}$

B $\dfrac{121}{243}$

C $\dfrac{3}{4}$

D $\dfrac{242}{243}$

E $\dfrac{121}{162}$

8. A rectangular array of real numbers has at least three rows and at least three columns. Consider the statement:

> Each row contains at least one entry which is greater than every other entry in its column.

Which of the following is the logical negation of this statement?

A There is a row such that, in every column, its entry is less than every entry in the other rows.

B There are two different rows such that, in every column, the entry in the first row is less than or equal to the entry in the second row.

C In every row, there is a column in which at least one other entry is greater than or equal to the entry in that row.

D There is a row such that, in every column, at least one other entry is greater than or equal to the entry in that row.

E There is a row such that, in every column, at least one other entry is strictly greater than the entry in that row.

F There is a row and a column in which at least one other entry is greater than or equal to the entry in that row.

9. Let a and b be positive real numbers. Consider the conditions

$$P: \quad a+b \text{ is rational}, \qquad Q: \quad a^2+b^2 \text{ is rational}.$$

Which of the following is correct?

A P is sufficient but not necessary for Q.

B P is both necessary and sufficient for Q.

C P is neither necessary nor sufficient for Q.

D P is necessary but not sufficient for Q.

10. In triangle ABC, the vertex A has coordinates $(0, 4)$ and M is the midpoint of AC.

The line BM has equation $y = x + 1$. The line through B perpendicular to AC has equation $y = 2x - 2$.

What are the coordinates of C?

A $(2, 3)$

B $(4, 2)$

C $\left(1, \frac{7}{2}\right)$

D $(6, 4)$

E $(4, 6)$

F $(12, 10)$

11. What is the sum of the squares of all the distinct real solutions of

$$\log_2(x^2 - 3x + 2) + \log_2(x^2 - 5x + 6) = 1?$$

A 4

B 8

C 10

D 18

E 12

12. Let a be a real number and let b be a positive real number. Consider the conditions

$$P: \quad a^2 + b^2 \leq 1,$$

$$Q: \quad ax + b \leq \sqrt{x^2 + 1} \quad \text{for every real number } x.$$

Which of the following is correct?

A P is both necessary and sufficient for Q.

B P is neither necessary nor sufficient for Q.

C P is sufficient but not necessary for Q.

D P is necessary but not sufficient for Q.

13. The graph $y = f(x)$, where f is a quadratic polynomial, lies above the x-axis for $0 \leq x \leq 2$. The area between the graph, the x-axis and the lines $x = 0$ and $x = 2$ is 9.

The trapezium rule with two strips of equal width gives an estimate of 10 for this area.

What is $f'(2) - f'(0)$?

A -12

B 3

C 6

D 12

E 24

14. A student attempts to prove that the equation $x^3 - x = 6$ has no positive real solution.

I. Suppose that $x > 0$ is a solution.

II. Dividing the equation by x gives $x^2 - 1 = 6x^{-1}$.

III. Differentiating both sides with respect to x gives $2x = -6x^{-2}$.

IV. The left-hand side is positive and the right-hand side is negative, which is a contradiction.

V. Therefore the equation has no positive real solution.

Which of the following best describes this attempt?

A The first error is in line I.

B The first error is in line II.

C The first error is in line III.

D The first error is in line IV.

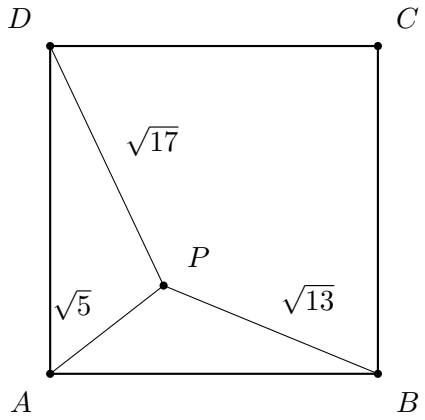
E The first error is in line V.

F The argument is correct.

15. The vertices of a square $ABCD$ are labelled in order. A point P lies strictly inside the square and satisfies

$$PA = \sqrt{5}, \qquad PB = \sqrt{13}, \qquad PD = \sqrt{17}.$$

What is the area of the square?



A 4

B $\sqrt{26}$

C 26

D 30

E 52

16. Two coins are tossed independently. Their probabilities of landing heads are p and q, where $0 < p < 1$ and $0 < q < 1$.

Consider the statements

P: $p \neq q$.

Q: The probability of obtaining exactly one head is greater than 1/2.

Is P necessary and/or sufficient for Q?

A P is necessary and sufficient for Q

B P is necessary but not sufficient for Q

C P is sufficient but not necessary for Q

D P is neither necessary nor sufficient for Q

17. What is the value of

$$\sum_{n=1}^{20} \frac{2n}{n^4+n^2+1}\,?$$

A $\frac{1}{421}$

B $\frac{210}{421}$

C $\frac{380}{381}$

D $\frac{400}{401}$

E $\frac{420}{421}$

F 1

18. A real-valued function f, defined for every real number, satisfies

$$f(x) + f(x+1) = 2x + 1$$

for every real x.

Consider the statements

I. $f(x+2) = f(x) + 2$ for every real x.

II. The values of f have no real upper bound.

III. If $a < b$, then $f(a) < f(b)$.

Which statements must be true?

A I only
B II only
C I and II only
D I and III only
E II and III only
F I, II and III

19. Thirty temperatures have a mean of 18°C.

When the temperatures are entered into a computer, ten are entered using their numerical values in degrees Fahrenheit. The other twenty are entered using their numerical values in degrees Celsius. The computer treats all thirty entries as degrees Celsius and reports a mean of 30°C.

The numerical Fahrenheit and Celsius values, F and C, are related by

$$F = \frac{9}{5}C + 32.$$

What is the mean, in degrees Celsius, of the ten temperatures entered incorrectly?

A 5

B $\frac{20}{9}$

C 18

D 41

E 37

20. Let $a_1, \ldots, a_n$ be real numbers, where $n \geq 2$ and $a_1 + \cdots + a_n > 0$.

A student wants to prove that some cyclic rearrangement of the list has every initial sum strictly positive. Put $S_0 = 0$ and $S_k = a_1 + \cdots + a_k$ for $1 \leq k \leq n$.

The student proposes the following proof.

1. Choose an index j from $0, \ldots, n-1$ by a rule still to be specified. Use the list
$$a_{j+1}, \ldots, a_n, a_1, \ldots, a_j.$$
2. An initial sum ending at a_k before reaching a_n is $S_k - S_j$, where $j < k < n$. It is strictly positive by the choice of j.
3. The sum ending at a_n is $S_n - S_j > 0$.
4. An initial sum continuing past a_n and ending at a_k, where $1 \leq k \leq j$, is E. It is strictly positive by the choice of j and the fact that $S_n > 0$.

Which rule for choosing j, together with which expression for E, makes this proof valid? In each option, minima and maxima are taken over $S_0, \ldots, S_{n-1}$.

A Choose the smallest j at which S_j is minimal; $E = S_n - S_j + S_k$

B Choose the largest j at which S_j is maximal; $E = S_n - S_j + S_k$

C Choose the largest j at which S_j is minimal; $E = S_n + S_j - S_k$

D Choose the largest j at which S_j is minimal; $E = S_n + S_k$

E Choose $j = 0$; $E = S_n - S_j + S_k$

F Choose the largest j at which S_j is minimal; $E = S_n - S_j + S_k$