1. The function f is defined by

$$f(x) = 2x^2 - 3x + 1.$$

A real number a satisfies $f(a+1) - f(a) = 9$.

What is the value of $f(a-1) - f(a)$?

A -13

B -9

C -5

D 5

E 13

2. A jug contains 900 mL of a drink made from concentrate and water in the volume ratio 1 : 4. The drink is mixed thoroughly, and 300 mL is poured out.

Water is then added to the jug until the volume ratio of concentrate to water is 1 : 9. Assume that volumes add when liquids are mixed.

How much water is added?

A 480 mL

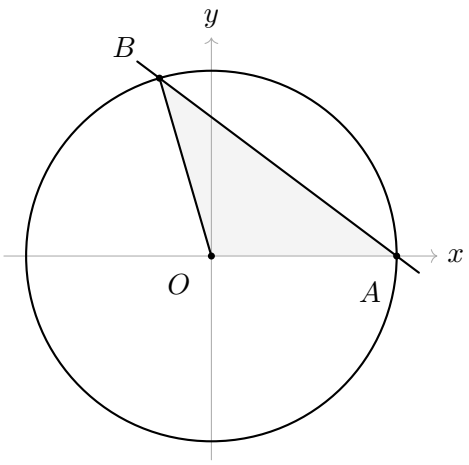
B 540 mL

C 600 mL

D 900 mL

E 1200 mL

3. The line $3x + 4y = 15$ meets the circle $x^2 + y^2 = 25$ at A and B. The point O is the origin. What is the area of triangle OAB?



A 6

B 9

C 12

D 15

E 24

F 25

4. In the expansion of

$$(1+x)^5(1+ax),$$

where a is a real constant, the coefficient of x^2 is zero.

What is the coefficient of x^3?

A -20

B -10

C 0

D 10

E 30

5. What is the sum of all the positive real solutions of

$$(x^2 - 3x)^2 = 2x^2 - 6x + 8\,?$$

A 3

B 4

C 6

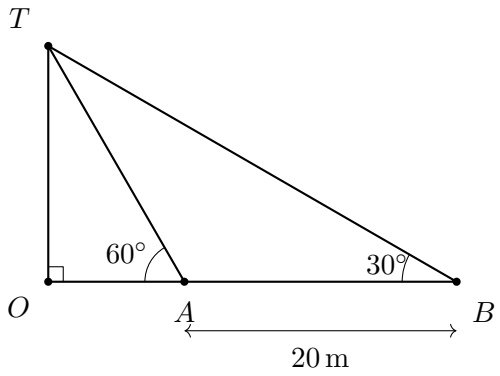
D 7

E 8

6. A drone is at a point T vertically above a point O on horizontal ground. Points A and B lie on a straight line on the ground, with A between O and B.

The angles of elevation of T from A and B are $60°$ and $30°$, respectively, and $AB = 20\,\text{m}$.

What is the height OT?



A $10\,\text{m}$

B $\dfrac{20\sqrt{3}}{3}\,\text{m}$

C $20\,\text{m}$

D $10\sqrt{3}\,\text{m}$

E $20\sqrt{3}\,\text{m}$

7. A cubic polynomial f satisfies

$$f'(0) = f'(4) = 9 \qquad \text{and} \qquad f(4) - f(0) = 4.$$

What is the positive difference between the x-coordinates of the two stationary points of $y = f(x)$?

A 1

B 2

C 3

D 4

E 6

8. Eight cards are numbered $1, 2, 3, \ldots, 8$. Two cards are selected at random without replacement, with every pair equally likely.

Given that the product of their numbers is divisible by 4, what is the probability that both numbers are even?

A $\frac{3}{14}$

B $\frac{3}{11}$

C $\frac{3}{7}$

D $\frac{1}{2}$

E $\frac{4}{7}$

9. The curve

$$y = \frac{(x-1)^2}{\sqrt{x}}, \qquad x > 0,$$

and the lines $x = 1$, $x = 4$ and $y = 0$ bound a region.

What is the area of this region?

A $\frac{38}{15}$

B $\frac{76}{15}$

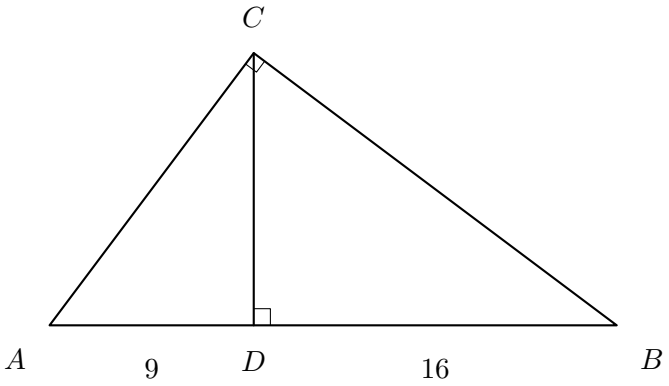
C $\frac{92}{15}$

D $\frac{16}{15}$

E $\frac{72}{5}$

10. Triangle ABC is right-angled at C. The perpendicular from C to AB meets AB at D, where $AD = 9$ and $DB = 16$.

What is the area of triangle ABC?



A 54

B 96

C 144

D 150

E 300

11. For which real values of k does the equation

$$x^2 - kx + k = 0$$

have two distinct real roots, both satisfying $0 < x < 4$?

A $0 < k < 4$

B $0 < k < \dfrac{16}{3}$

C $4 \leq k < \dfrac{16}{3}$

D $4 < k \leq \dfrac{16}{3}$

E $4 < k < \dfrac{16}{3}$

F $k > 4$

12. A geometric progression has positive terms $u_1, u_2, u_3, \ldots$ and satisfies

$$u_1 + u_2 + u_3 = 21, \qquad u_4 + u_5 + u_6 = 168.$$

What is the value of $u_1^2 + u_2^2 + u_3^2$?

A 189

B 21

C 63

D 147

E 441

13. The angles α and β satisfy

$$0 < \alpha < \beta < \frac{\pi}{2},$$

$$\sin\alpha + \cos\alpha = \sin\beta + \cos\beta, \qquad \tan\alpha + \tan\beta = 4.$$

What is the value of $\sin\beta - \sin\alpha$?

A $-\dfrac{\sqrt{2}}{2}$

B 0

C $\dfrac{1}{4}$

D $\dfrac{1}{2}$

E $\dfrac{\sqrt{6}}{2}$

F $\dfrac{\sqrt{2}}{2}$

14. The length and width of a rectangle, each rounded to the nearest centimetre, are 12 cm and 8 cm respectively.

What is the least integer N for which the rectangle's area is certainly less than N cm^2?

A 96

B 107

C 100

D 104

E 106

15. The function f is defined by

$$f(x) = \frac{x^2 + 2x + 5}{x^2 + 2x + 2}, \qquad -3 < x < 0.$$

Which inequality describes the complete set of values of $y = f(x)$?

A $1 < y \leq 4$

B $\frac{8}{5} \leq y \leq 4$

C $\frac{8}{5} < y < 4$

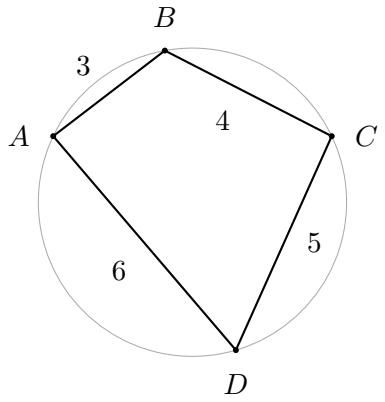
D $\frac{8}{5} < y \leq 4$

E $\frac{5}{2} < y \leq 4$

16. The vertices A, B, C and D of a convex quadrilateral lie, in that order, on a circle. Its side lengths are

$$AB = 3, \qquad BC = 4, \qquad CD = 5, \qquad DA = 6.$$

What is the area of $ABCD$?



A 9

B $6\sqrt{10}$

C $3\sqrt{10}$

D $6\sqrt{7}$

E 21

F $12\sqrt{10}$

17. A histogram represents journey times t, in minutes. Its classes and frequency densities are shown below, where $k > 0$.

Journey time	$0 \leq t < 10$	$10 \leq t < 30$	$30 \leq t < 40$
Frequency density	2	k	3

The estimate of the mean calculated using the class midpoints is 22 minutes.

What is the estimated median, in minutes, obtained by linear interpolation at half the total frequency?

A 17

B 20

C 22

D 24

E 40

18. During the first 3 minutes after a tap is opened, the volume of water in a tank is modelled by

$$V(t) = t^3 + 3,$$

where V is measured in litres and t in minutes.

At $t = a$, the instantaneous rate of increase of the volume equals the average rate of increase over those first 3 minutes.

What is a?

A 1

B $\frac{3}{2}$

C $\sqrt{3}$

D $\sqrt{\frac{10}{3}}$

E 3

19. The numbers $a_0, a_1, \ldots, a_9$ are defined by

$$(1+x)^9 = a_0 + a_1 x + \cdots + a_9 x^9.$$

Which of the following is equal to

$$a_0 a_1 + a_1 a_2 + \cdots + a_8 a_9 \,?$$

A $\binom{9}{4}\binom{9}{5}$

B $\binom{18}{7}$

C $\binom{18}{9}$

D $2\binom{18}{8}$

E 2^{17}

F $\binom{18}{8}$

20. The real numbers x and y satisfy

$$x^2 + y^2 = 13, \qquad x^3 + y^3 = 35.$$

What is the sum of all the distinct possible values of xy?

A -27

B $-\frac{9}{2}$

C $\frac{3}{2}$

D 3

E 6

F $\frac{39}{2}$