1. The equations

$$x^2 + ax + 12 = 0 \qquad \text{and} \qquad x^2 + (a+3)x + 3 = 0$$

have a common real solution. What is the value of a?

A -7

B -4

C 1

D 3

E 7

2. Six real numbers $a \leq b \leq c \leq d \leq e \leq f$ have mean 10. The mean of a, b, c, d is 7, and the mean of c, d, e, f is 12. What is the median of the six numbers?

A 7

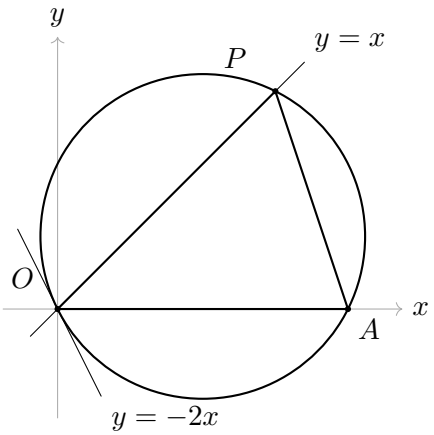
B 8

C $\frac{19}{2}$

D 10

E 16

3. A circle passes through $O = (0, 0)$ and $A = (8, 0)$. The tangent to the circle at O is the line $y = -2x$. The line $y = x$ meets the circle again at P. What is the area of triangle OAP?



A 8

B 12

C 16

D 20

E 24

4. The tangent to the curve

$$y = x^3 + ax^2$$

at the point where $x = 2$ has gradient 4. What is the y-intercept of this tangent?

A -16

B -8

C -4

D 0

E 8

5. For every positive integer n, the sum of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is

$$2n^2 + 3n.$$

What is the least value of n for which $u_n > 101$?

A 7

B 25

C 26

D 50

E 51

6. The equation

$$2\cos^2\theta = 3\sin\theta\cos\theta$$

has four distinct solutions $\theta_1, \theta_2, \theta_3, \theta_4$ with $0 \leq \theta < 2\pi$. What is the value of

$$\sin^2\theta_1 + \sin^2\theta_2 + \sin^2\theta_3 + \sin^2\theta_4\,?$$

A $\dfrac{4}{13}$

B $\dfrac{8}{13}$

C $\dfrac{21}{13}$

D $\dfrac{34}{13}$

E 4

7. A function f, defined for all real numbers, satisfies

$$f(x) + 2f(1-x) = x^2$$

for every real x. For which values of the real constant k does the equation

$$f(|x|) = k$$

have exactly four distinct real solutions?

A $k > -\frac{2}{3}$

B $k \geq \frac{2}{3}$

C $-\frac{2}{3} \leq k < \frac{2}{3}$

D $-\frac{2}{3} < k \leq \frac{2}{3}$

E $0 < k < \frac{2}{3}$

F $-\frac{2}{3} < k < \frac{2}{3}$

8. A fair six-sided die, numbered 1 to 6, is rolled. If the result is 1, 2 or 3, the die is rolled once more and the score is the sum of the two results. Otherwise, there is no second roll and the score is the first result.

Given that the score is at least 6, what is the probability that the die was rolled twice?

A $\frac{1}{4}$

B $\frac{1}{3}$

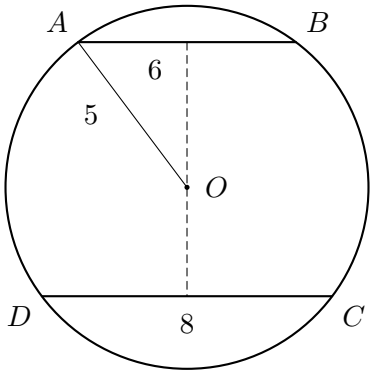
C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{9}{10}$

9. A circle of radius 5 has parallel chords AB and CD of lengths 6 and 8, respectively. The chords lie on opposite sides of the centre O.

What is the perpendicular distance between the chords?



A 1

B 3

C 4

D 5

E 7

F 14

10. The curve $y = f(x)$ satisfies

$$f'(x) = 6x - 4.$$

The normal to the curve at the point with x-coordinate 1 passes through $(0, 5)$.

What is the value of

$$\int_0^2 f(x)\,\mathrm{d}x\,?$$

A 8

B 9

C 11

D 13

E 16

11. Nonzero real numbers x, y and z satisfy

$$x + y + z = 0, \qquad x^2 + y^2 + z^2 = 14, \qquad x^3 + y^3 + z^3 = -18.$$

What is the value of

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\,?$$

A $\frac{7}{6}$

B $-\frac{7}{6}$

C $\frac{7}{3}$

D $-\frac{7}{3}$

E $\frac{6}{7}$

F $-\frac{6}{7}$

12. What is the coefficient of x^5 in the expansion of

$$(1-x)^4(1+x)^7\,?$$

A -18

B -14

C -4

D 4

E 14

F 18

13. A point P moves at constant speed around a fixed vertical circle, completing one revolution in 36 seconds. The greatest and least heights of P above horizontal ground are 18 metres and 2 metres, respectively.

During one complete revolution, for how many seconds is P more than 14 metres above the ground?

A 6

B 9

C 12

D 18

E 24

14. What is the least positive integer N for which $N/12$ is the square of an integer and $N/18$ is the cube of an integer?

A $2^2 3^3$

B $2^4 3^5$

C $2^4 3^3$

D $2^2 3^5$

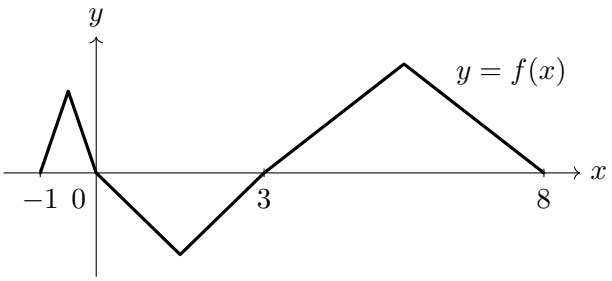
E $2^5 3^4$

F $2^6 3^6$

15. The graph of $y = f(x)$ is shown for $-1 \leq x \leq 8$, which is the domain of f. Its only zeros are -1, 0, 3 and 8.

What is the total length of the set of real values of x for which

$$f(x^2 - 1) > 0\,?$$



A 4

B 2

C $1 + \sqrt{5}$

D 6

E $2 + 2\sqrt{5}$

16. The circle C has equation

$$(x-3)^2 + (y-3)^2 = 1.$$

Consider all circles whose centres lie in the first quadrant and which are tangent to both coordinate axes and to C.

What is the sum of the radii of these circles?

A 4

B 10

C 12

D 14

E 24

F 48

17. A bag contains three red counters and two blue counters. A counter is chosen at random.

If it is red, it is returned to the bag. If it is blue, it is not returned.

A second counter is then chosen at random from those in the bag. What is the probability that the two counters chosen have different colours?

A $\frac{6}{25}$

B $\frac{3}{10}$

C $\frac{12}{25}$

D $\frac{27}{50}$

E $\frac{3}{5}$

18. A particle moves on a straight line. At time t seconds its coordinate, in metres, is

$$x(t) = t(t-2)(t-5).$$

For how many seconds in total during $0 < t < 5$ is the particle moving towards the origin?

A 2

B $\frac{7}{3}$

C $\frac{5}{2}$

D $\frac{8}{3}$

E $\frac{2\sqrt{19}}{3}$

F 3

19. A sequence of positive numbers is defined by

$$u_1 = \frac{1}{2}, \qquad u_{k+1} = \frac{u_k}{1+u_k} \quad (k \geq 1).$$

For a positive integer n,

$$u_n u_{n+1} = \frac{1}{156}.$$

What is the value of

$$\frac{1}{u_1} + \frac{1}{u_2} + \cdots + \frac{1}{u_n}\,?$$

A 66

B 76

C 77

D 78

E 90

F 156

20. For some real constants a and b, the polynomial

$$x^4 + ax^3 + (a+3)x^2 + bx + 4$$

is the square of a quadratic polynomial with real coefficients.

What is the sum of all the distinct possible values of b?

A -12

B -8

C 0

D 4

E 8

F -4