Cambridge Assessment Admissions Testing

# TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

## PAPER 2

## SPECIMEN

Time: 75 minutes

Additional Materials: Answer sheet

### INSTRUCTIONS TO CANDIDATES

**Please read these instructions carefully, but do not open the question paper until you are told that you may do so.**

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the second of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators must **NOT** be used. There is no formulae booklet for this test.

**Please wait to be told you may begin before turning this page**

This question paper consists of 15 printed pages and 4 blank pages

tmua.me editorial edition: corrections applied; original question numbering retained.
20 active questions.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

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**1.** The radius of the circle $2x^2 + 2y^2 - 8x + 12y + 15 = 0$ is

**A** $\sqrt{\frac{5}{2}}$

**B** $\sqrt{\frac{11}{2}}$

**C** $\sqrt{\frac{41}{2}}$

**D** $\sqrt{37}$

**E** $\sqrt{67}$

**2.** The gradient of the curve $y = \frac{(3x-2)^2}{x\sqrt{x}}$ at the point where $x = 2$ is

**A** $\frac{3}{2}\sqrt{2}$

**B** $3\sqrt{2}$

**C** $4\sqrt{2}$

**D** $\frac{9}{2}\sqrt{2}$

**E** $6\sqrt{2}$

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**3.** Consider the following attempt to solve an equation. The steps have been numbered for reference.

$$\sqrt{x+5} = x+3$$

(1)

$$x+5 = x^2+6x+9$$

(2)

$$x^2+5x+4 = 0$$

(3)

$$(x+4)(x+1) = 0$$

$$x = -4 \text{ or } x = -1$$

Which one of the following statements is true?

**A** Both $-4$ and $-1$ are solutions of the equation.

**B** Neither $-4$ nor $-1$ are solutions of the equation.

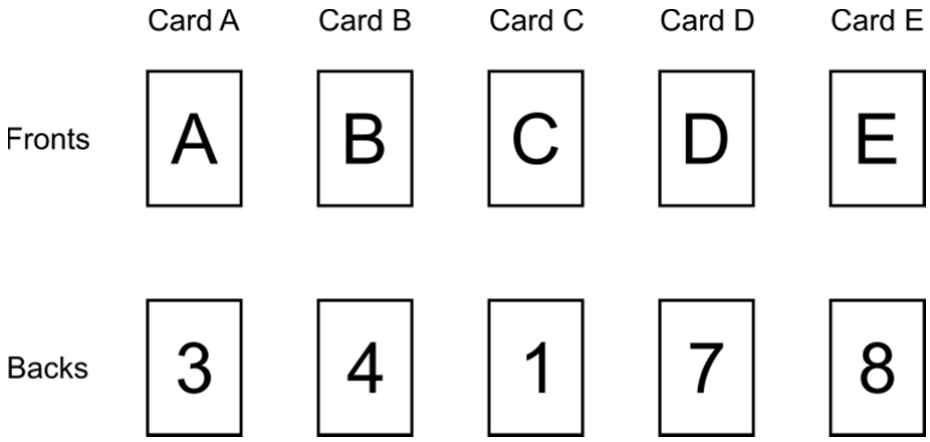
**C** One solution is correct and the incorrect solution arises as a result of step (1).

**D** One solution is correct and the incorrect solution arises as a result of step (2).

**E** One solution is correct and the incorrect solution arises as a result of step (3).

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**4.** A set of five cards each have a letter printed on their front and a number printed on their back, as follows:




Which one of the five cards (**A**, **B**, **C**, **D** or **E**) provides a counterexample to the following statement?

*Every card that has a vowel on its front has an even number on its back.*

**5.** Using the observation that $2^5 \approx 3^3$, it is possible to deduce that $\log_3 2$ is approximately

**A** $\frac{3}{5}$

**B** $\frac{2}{3}$

**C** $\frac{3}{2}$

**D** $\frac{5}{3}$

**E** $\frac{1}{2}$

**F** $2$

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## Question 6

The area of a rectangle is measured to be $5600\,\text{cm}^2$ correct to 2 significant figures.

The width of the rectangle is measured to be $80\,\text{cm}$ correct to the nearest centimetre.

Which one of the following expressions gives the least upper bound for the height of the rectangle?

**A** $70.5\,\text{cm}$

**B** $75\,\text{cm}$

**C** $\dfrac{5650}{85}\,\text{cm}$

**D** $\dfrac{5650}{80.5}\,\text{cm}$

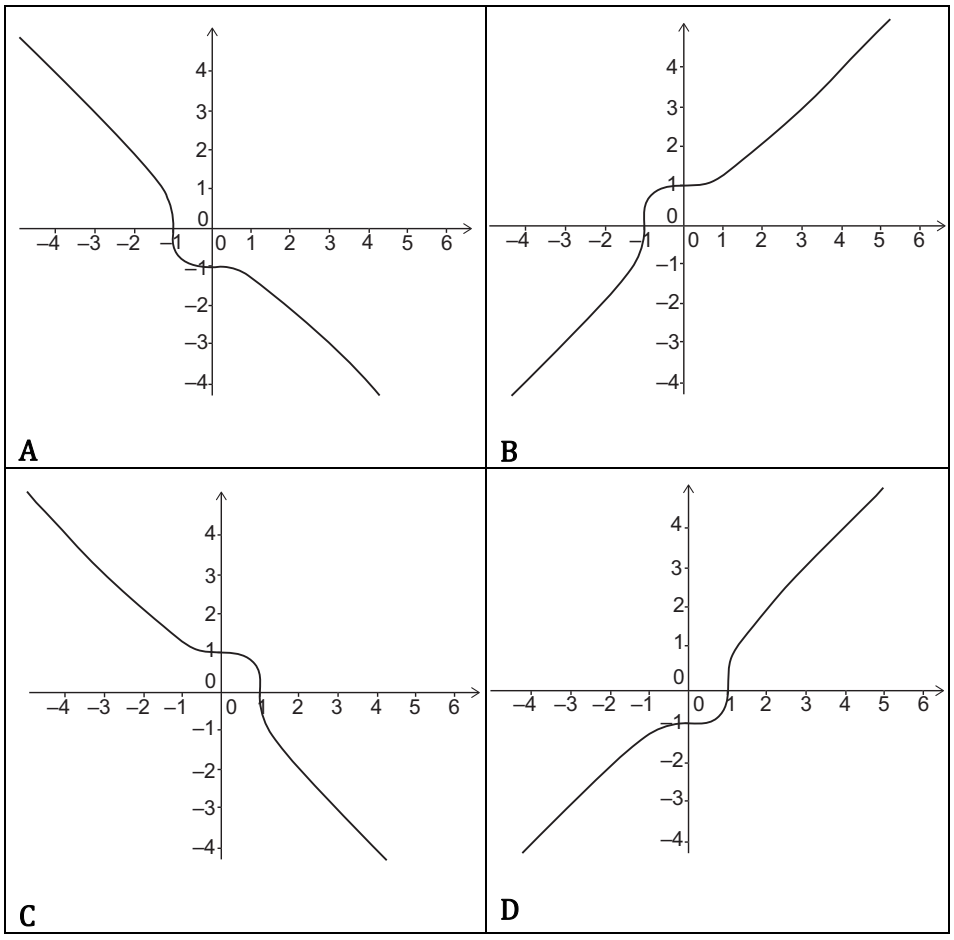
**E** $\dfrac{5650}{75}\,\text{cm}$

**F** $\dfrac{5650}{79.5}\,\text{cm}$

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p057-q006.

**7.** Which one of the following is a sketch of the graph

$$(x + y)(x^2 - xy + y^2) = 1\,?$$




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**8.** Consider the following statement about the positive integer $n$:

Statement (*): *The sum of the four consecutive integers, the smallest of which is $n$, is a multiple of 6.*

Which one of the following is true?

**A** Statement (*) is true for all values of $n$.

**B** Statement (*) is true for all values of $n$ which are odd, but not for any other values of $n$.

**C** Statement (*) is true for all values of $n$ which are multiples of 3, but not for any other values of $n$.

**D** Statement (*) is true for all values of $n$ which are multiples of 6, but not for any other values of $n$.

**E** Statement (*) is not true for any value of $n$.

**9.** Consider the statement about Fred:

(*) *Every day next week, Fred will do at least one maths problem.*

If statement (*) is **not** true, which of the following is **certainly** true?

**A** Every day next week, Fred will do more than one maths problem.

**B** Some day next week, Fred will do more than one maths problem.

**C** On no day next week will Fred do more than one maths problem.

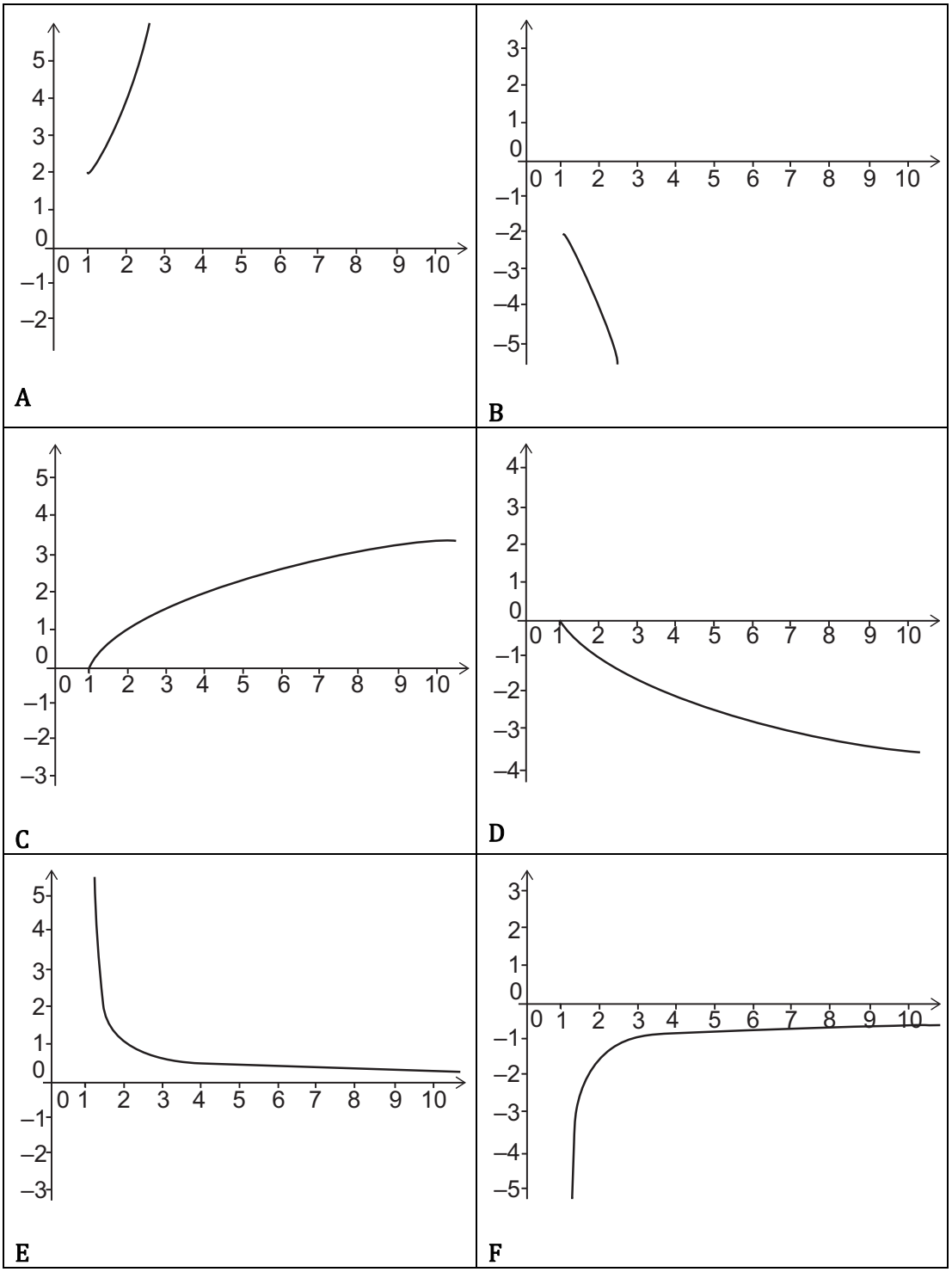
**D** Every day next week, Fred will do no maths problems.

**E** Some day next week, Fred will do no maths problems.

**F** On no day next week will Fred do no maths problems.

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**10.** Which one of the following is a sketch of the graph of $y = \log_x 2$ for $x > 1$?




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**11.** Which one of the following numbers is largest in value?
(All angles are given in radians.)

**A** $\tan\left(\frac{3\pi}{4}\right)$

**B** $\log_{10} 100$

**C** $\sin^{10}\left(\frac{\pi}{2}\right)$

**D** $\log_2 10$

**E** $\left(\sqrt{2}-1\right)^{10}$

**12.** A polynomial $p(x)$ has the property that $p(1) = 2$.

Which one of the following can be deduced from this?

**A** $p(x) = (x-1)q(x) + 2$ for some polynomial $q(x)$.

**B** $p(x) = (x+1)q(x) + 2$ for some polynomial $q(x)$.

**C** $p(x) = (x-1)q(x) - 2$ for some polynomial $q(x)$.

**D** $p(x) = (x+1)q(x) - 2$ for some polynomial $q(x)$.

**E** $p(x) = (x-2)q(x) + 1$ for some polynomial $q(x)$.

**F** $p(x) = (x+2)q(x) + 1$ for some polynomial $q(x)$.

**G** $p(x) = (x-2)q(x) - 1$ for some polynomial $q(x)$.

**H** $p(x) = (x+2)q(x) - 1$ for some polynomial $q(x)$.

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**13.** Five runners competed in a race: Fred, George, Hermione, Lavender, and Ron.

Fred beat George.

Hermione beat Lavender.

Lavender beat George.

Ron beat George.

Assuming there were no ties, how many possible finishing orders could there have been, given only this information?

**A** 1

**B** 6

**C** 12

**D** 18

**E** 24

**F** 120

**14.** The graph of the polynomial function

$$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f,$$

is sketched, where $a$, $b$, $c$, $d$, $e$, and $f$ are real constants with $a \neq 0$.

Which one of the following is **not** possible?

**A** The graph has two local minima and two local maxima.

**B** The graph has one local minimum and two local maxima.

**C** The graph has one local minimum and one local maximum.

**D** The graph has no local minima or local maxima.

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**15.** For any real numbers $a$, $b$, and $c$ where $a \geq b$, consider these three statements:

**1** $-b \geq -a$

**2** $a^2 + b^2 \geq 2ab$

**3** $ac \geq bc$

Which of the statements 1, 2, and 3 **must** be true?

**A** none

**B** 1 only

**C** 2 only

**D** 3 only

**E** 1 and 2 only

**F** 1 and 3 only

**G** 2 and 3 only

**H** 1,2 and 3

**16.** The sequence $a_n$ is given by the rule:

$$a_1 = 2$$

$$a_{n+1} = a_n + (-1)^n \text{ for } n \geq 1$$

What is

$$\sum_{n=1}^{100} a_n$$

**A** 150

**B** 250

**C** $-4750$

**D** 5150

**E** $4\left(1 - \left(\frac{1}{2}\right)^{100}\right)$

**F** $4\left(\left(\frac{3}{2}\right)^{100} - 1\right)$

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**17.** Let $S$ be a set of positive integers, for example $S$ could consist of 3, 4, and 8.

A positive integer $n$ is called an $S$-number **if and only if** for every factor $m$ of $n$ with $m > 1$, the number $m$ is a multiple of some number in $S$.

So in the above example, 9 is an $S$-number; this is because the factors of 9 greater than 1 are 3 and 9, and each of these is a multiple of 3.

Positive integer $n$ is therefore **not** an $S$-number **if and only if**

**A** for **every** (positive) factor $m$ of $n$ with $m > 1$,
there is a number in $S$ which is not a factor of $m$.

**B** for **every** (positive) factor $m$ of $n$ with $m > 1$,
there is no number in $S$ which is a factor of $m$.

**C** for **every** (positive) factor $m$ of $n$ with $m > 1$,
every number in $S$ is a factor of $m$.

**D** for **some** (positive) factor $m$ of $n$ with $m > 1$,
there is a number in $S$ which is not a factor of $m$.

**E** for **some** (positive) factor $m$ of $n$ with $m > 1$,
there is no number in $S$ which is a factor of $m$.

**F** for **some** (positive) factor $m$ of $n$ with $m > 1$,
every number in $S$ is a factor of $m$.

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**18.** A group of five numbers are such that:

- their mean is 0
- their range is 20

What is the largest possible median of the five numbers?

**A** 0

**B** 4

**C** $4\frac{1}{2}$

**D** $6\frac{1}{2}$

**E** 8

**F** 20

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**19.** The positive real numbers $a$, $b$, and $c$ are such that the equation

$$x^3 + ax^2 = bx + c$$

has three real roots, one positive and two negative.

Which one of the following correctly describes the real roots of the equation

$$x^3 + c = ax^2 + bx \, ?$$

**A** It has three real roots, one positive and two negative.

**B** It has three real roots, two positive and one negative.

**C** It has three real roots, but their signs differ depending on $a$, $b$, and $c$.

**D** It has exactly one real root, which is positive.

**E** It has exactly one real root, which is negative.

**F** It has exactly one real root, whose sign differs depending on $a$, $b$, and $c$.

**G** The number of real roots can be one or three, but the number of roots differs depending on $a$, $b$, and $c$.

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**20.** Five logicians each make a statement, as follows:

**Mr P**: Of these five statements, an odd number are true.

**Ms Q**: Both statements made by women are true.

**Mr R**: My first name is Robert and Mr P's statement is true.

**Ms S**: Exactly one statement made by a man is true.

**Mr T**: Neither statement made by a woman is true.

How many of the five statements can be simultaneously true?

**A** none

**B** 1 only

**C** 2 only

**D** 3 only

**E** 4 only

**F** none or 1 only

**G** 1 or 2 only

**H** 2 or 3 only

**END OF TEST**

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