TMUA

COMMUNITY PAPERS

Zach

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Zach/Cody

tmua.me editorial edition: corrections applied; original question numbering retained.
20 active questions.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

Question 1

Let a_n be the area bounded by the curves $x = y$, $x^2 + y^2 = n$ and the positive x-axis. What is $\sum_{n=1}^{100} a_n$?

A $\frac{250\pi}{2}$

B $\frac{50\pi}{4}$

C $\frac{5050\pi}{8}$

D $\frac{3805\pi}{8}$

E $150\sqrt{2}\pi$

Question 2

The symbol $\{x\}$ denotes the fractional part of x. For example $\{10.4\} = 0.4$, $\{2\} = 0$. Given that $f(x) = 5x^3 + 2x^2 + 8x + 1$, find

$$\int_0^n f(\{x\} + 1)\,\mathrm{d}x$$

where n is a positive integer.

A $\frac{425n}{13}$

B $\frac{1}{2}n(n+1)$

C $\frac{83n}{12}$

D $\frac{437n}{12}$

E $15n^2 + 4n + 8$

Question 3

Let x and y be real numbers such that $x^2 + y^2 = 49$. What is the greatest value of $12x - 5y$?

A 91

B 84

C 104

D 89

E 87

Question 4

For what values of x is

$$x + 3 = 1 + 1 + x + 1 + 2x + x^2 + 1 + 3x + 3x^2 + x^3 + ...$$

A -1

B 0

C $\frac{-3-\sqrt{5}}{2}$

D $\frac{-3+\sqrt{5}}{2}$

E $\frac{-3-\sqrt{5}}{2}$ and $\frac{-3+\sqrt{5}}{2}$

Question 5

How many four-digit square numbers are multiples of 3, 4 and 5?

A 0

B 1

C 2

D 3

E 4

F 5

G 6

Question 6

Two identical ice sculptures are left to melt outside. Sculpture A melts in 6 hours. Sculpture B melts in 12 hours. Assuming each sculpture melts uniformly, after how long will Sculpture B have twice as much ice remaining as Sculpture A?

A 3 hours

B 3.5 hours

C 4 hours

D 4.5 hours

E 5 hours

Question 7

Alvin and Simon can create a song in 5 days. Alvin and Theodore can create a song in 8 days. Simon and Theodore can create a song in 10 days. Approximately how many days does it take Alvin, Simon and Theodore to make a song together?

A 3

B 3.2

C 3.5

D 3.7

E 4.7

Question 8

Find x such that

$$\ln(x\ln(x\ln(x\ln(x\ln(x\cdots = e$$

A e

B e^3

C e^{e+1}

D e^e

E e^{e-1}

Question 9

Find the coefficient of x^{15} in the expansion of $(2x^3 + 3x + 4x^5)^5$.

A 11280

B 9920

C 8462

D 12482

E 10592

Question 10

Find the units digit of $34657^{267} + 81$.

A 0

B 1

C 2

D 3

E 4

F 5

G 6

H 7

I 8

J 9

Question 11

There are 129 coins on a table. 128 are fair, and one has heads on both sides. A coin is tossed 8 times and lands on heads every time. What is the probability it lands on heads if thrown a 9th time?

A $\frac{1}{2}$

B $\frac{9}{10}$

C $\frac{7}{8}$

D $\frac{5}{6}$

E $\frac{2}{3}$

Question 12

Find the sum of the solutions to

$$16^{\log_2 x} - 10 \times 27^{\log_3 x} + 35 \times 64^{\log_8 x} - 50x + 24 = 0$$

A 0

B 8

C 10

D 24

E 43

Question 13

How many solutions does

$$\sin x + \sin^2 x + \sin^3 x + \cdots = \tan x$$

have for $0 \leq x \leq 2\pi$?

A 1

B 2

C 3

D 4

E 5

F 6

Question 14

How many sets of positive integers (a, b, c) with $a \leq b \leq c$ satisfy

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1?$$

A 1

B 2

C 3

D 4

E 0

F ∞

Question 15

For $x \geq 0$, $x \neq 2$, let

$$f(x) = \frac{x\sqrt{x} - 2\sqrt{x}}{\sqrt{2} - \sqrt{x}}.$$

Define $f(2)$ by continuous extension. Find $f\prime(2)$.

A 0

B $\frac{1}{\sqrt{2}}$

C $\frac{1}{2}$

D $-\frac{3}{2}$

E $\frac{3}{2}$

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p104-q015.

Question 16

A sheep called Shaun lives in a circular pen of radius 8. His position inside the pen is random. What is the probability that Shaun is within 1 m of a side of the pen?

A $\frac{1}{2}$

B $\frac{49}{64}$

C $\frac{7}{8}$

D $\frac{15}{64}$

E $\frac{1}{8}$

Question 17

Solve:

$$\log_{x-2} 9 = \log_x 81$$

A $x = 1$

B $x = 4$

C $x = 1$ and $x = 4$

D No real solutions

E All of the above

Question 18

For which function is

$$f(f(f(f(f(x))))) \neq 0?$$

A $f(x) = \sin x$

B $f(x) = x^7$

C $f(x) = e^x - 1$

D $f(x) = \cos^2 x$

Question 19

A proper divisor of a number is a positive factor that is smaller than the number.

E.g. The proper divisors of 6 are 1,2,3

How many proper divisors does 30^4 have?

A 63

B 125

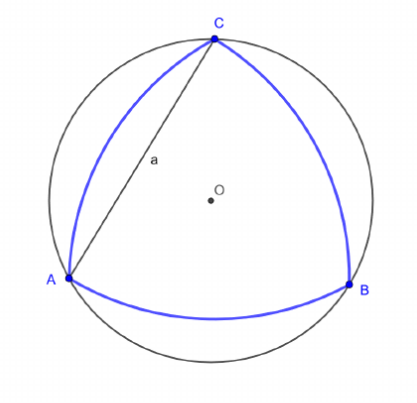
C 124

D 64

E None of the above

Question 20

Given that CB is an arc formed from the circle of radius a centred at A, and AB and AC are defined similarly, what is the area of the shape ABC in terms of a?



A $\frac{a^2}{2}\left(\pi - 3\tan\frac{\pi}{6}\right)$

B $\frac{a^2}{2}\left(\pi - 3\tan\frac{\pi}{3}\right)$

C $\frac{a^2}{2}\left(\pi - 3\sin\frac{\pi}{6}\right)$

D $\frac{a^2}{2}\left(\pi - 3\sin\frac{\pi}{3}\right)$

E $3a^2\pi\tan\left(\frac{\pi}{6}\right)$