TMUA

COMMUNITY PAPERS

Proven Potential TMUA Sample Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Proven Potential

tmua.me editorial edition: corrections applied; original question numbering retained.
19 active questions. Withdrawn question(s): 7.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

Question 1

The graph $y = 3x^2 - 15x + 18$ is plotted on a set of axes. What is the turning point of the graph?

A It is a point of maxima at $\left(\frac{5}{2}, -\frac{51}{4}\right)$

B It is a point of minima at $\left(\frac{5}{2}, -\frac{51}{4}\right)$

C It is a point of maxima at $\left(\frac{5}{2}, -\frac{3}{4}\right)$

D It is a point of minima at $\left(\frac{5}{2}, -\frac{3}{4}\right)$

E It is a point of maxima at $\left(\frac{5}{2}, -\frac{1}{4}\right)$

F It is a point of minima at $\left(\frac{5}{2}, -\frac{1}{4}\right)$

G It is a point of maxima at $\left(\frac{5}{2}, \frac{147}{4}\right)$

H It is a point of minima at $\left(\frac{5}{2}, \frac{147}{4}\right)$

Question 2

The function $f(x)$ is defined piecewise by

$$f(x) = \begin{cases} x^2, & 0 \leq x < \sqrt[3]{k+1}, \\ \sqrt{x}, & \sqrt[3]{k+1} \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

The probability that the variable lies between two values of x is given by the area under the graph of $f(x)$ between those values. Given that the total probability is 1, find the value of k.

A $k = 0$

B $k = 2 + 2\sqrt{2}$

C $k = 2 - 2\sqrt{2}$

D $k = \sqrt{2}$

E $k = -\sqrt{2}$

F $k = 8$

G $k = 2$

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p102-q002.

Question 3

Three fair dice labelled 1 to 6 are rolled together, what is the probability that the numbers rolled could be the sides of an equilateral triangle?

A $\frac{23}{72}$

B $\frac{1}{216}$

C $\frac{1}{36}$

D $\frac{1}{6}$

E $\frac{5}{54}$

Question 4

How many solutions of x are there to the equation:

$$\sin(-x)\cos(x) = \sin(x)\cos(-x) + 0.5 \qquad 0 \leq x \leq 2\pi$$

A 4

B 0

C 2

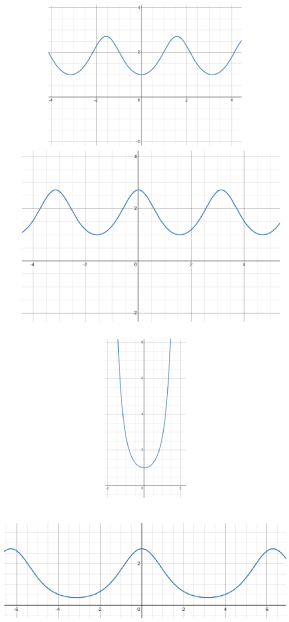
D 5

E 8

F 3

Question 5

Which is the graph of $y = e^{\cos^2 x}$?



A Graph A

B Graph B

C Graph C

D Graph D

Question 6

ABC is a triangle.

$$AB = 5$$
$$\widehat{CAB} = 30^\circ$$
$$BC < 5$$

What length of BC defines ABC uniquely?

A 2.5

B 5

C 2

D 1

E 3

Question 8

How many solutions are there to the equation below?

$$|x| + |y| \leq 10 \qquad x,\ y \text{ are integers}$$

A 200

B 100

C 40

D 121

E 181

F 66

G 221

Question 9

You are given that

$$S = 1 + \log_3 \frac{1}{3} + \log_3 \frac{1}{9} + \log_3 \frac{1}{27} + \cdots + \log_3 \frac{1}{3^{n-1}} + \log_3 \frac{1}{3^n}$$

If $S = -2$, what is the value of n?

A 3

B -3

C 2

D 6

E No solutions

Question 10

You are given the equation

$$(k + (1 + x)^3)^5, \qquad \text{where } |k| < 100$$

The coefficient of x^9 is 2845, what is the coefficient of x^7?

A 6435

B -6435

C 1440

D -1440

E 45

F -45

G 7920

H -7920

Question 11

Three sides of a right angle triangle are of a geometric progression, what is the hypotenuse of the triangle in terms of the shortest length a, where the common ratio $r > 1$?

A $\frac{1+\sqrt{5}}{2}a$

B $\frac{1-\sqrt{5}}{2}a$

C $\sqrt{\frac{1+\sqrt{5}}{2}}a$

D $\frac{\sqrt{5}-1}{2}a$

E $\sqrt{\frac{\sqrt{5}-1}{2}}a$

Question 12

Estimate the number of solutions to the equation

$$10^{12} \sin x = 1 - x^2$$

A 10^2

B 10^3

C 10^4

D 10^5

E 10^6

F 10^{12}

Question 13

Two players, A and B, play a series of games. Each game is independent, and player A has a probability $p = \frac{2}{3}$ of winning any single game.

The match ends as soon as player A wins two games in a row. Find the probability that player A wins the match within the first n games, where n is a positive integer.

A $1 - \frac{4}{3}\left(\frac{2}{3}\right)^n + \frac{1}{3}\left(-\frac{1}{3}\right)^n$

B $1 - \left(1 - \frac{4}{9}\right)^n$

C $1 - \left(\frac{1}{3}\right)^n$

D $\frac{4}{9}$

Question 14

What is the value of

$$2 + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} + \cdots + \binom{n}{n-2} + \binom{n}{n-1}$$

A $\frac{n(n+1)}{2}$

B 2^n

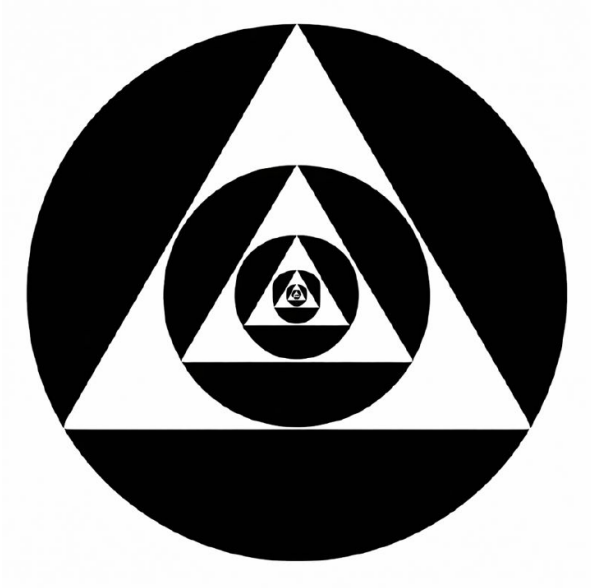
C $2^n - n$

D n^n

E $2^n - 1$

Question 15

If the largest circle has radius 1, what is the area of the shaded region?



A $\frac{4\pi}{3} - \sqrt{3}$

B $\frac{4\pi}{3}$

C $\frac{2\pi}{3} - \sqrt{3}$

D $\pi - \frac{3\sqrt{3}}{4}$

E $\pi - \sqrt{3}$

F No solution

Question 16

A sequence is defined by

$$a_{n+1} = \frac{1}{1 + a_n}, \qquad a_1 = 1$$

What is the value of a_n as $n \to \infty$?

A $\frac{1}{\sqrt{5}}$

B 1

C $\frac{1+\sqrt{5}}{2}$

D $\frac{\sqrt{5}-1}{2}$

E $\frac{1-\sqrt{5}}{2}$

Question 17

How many solutions are there to the equation

$$\sqrt{x+3}+\sqrt{7-x}=4$$

A 0

B 1

C 2

D 3

E 4

F 5

Question 18

What is the minimum value of the function $f(x) = |x - 1| + |2x + 3|$?

A 1

B 2.5

C 2

D 4

E 5

F 1.5

Question 19

Two concentric circles have radii r and $2r$. A chord of the larger circle is tangent to the smaller circle. Find the length of the chord in terms of r.

A $2r$

B $4r$

C $2r\sqrt{2}$

D $r\sqrt{3}$

E $3r$

F $r\sqrt{2}$

G $6r$

H $2r\sqrt{3}$

I Not enough information

Question 20

How many solutions are there to the equation

$$x^{x^2} = x^4$$

A 0

B 1

C 2

D 3

E 4

F 5

G Infinite