TMUA

COMMUNITY PAPERS

James Chen TMUA Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: James Chen

tmua.me editorial edition: corrections applied; original question numbering retained.
20 active questions.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

Question 1

What is the gradient of the perpendicular bisector of the line segment connecting the points $(-3, 5)$ and $(-1, 9)$?

A -0.5

B 0.5

C -2

D 2

Question 2

How many distinct real solutions does the equation $x^8 = 2x^4 + 4$ have?

A 0

B 1

C 2

D 4

E 6

F 8

Question 3

If $\pi^2 \approx g$, $\pi > 0$, then which estimate for $\ln(\pi)$ can be deduced?

A $\ln(\pi) \approx \frac{g}{2}$

B $\ln(\pi) \approx \ln\left(\sqrt{g}\right)$

C $\ln(\pi) \approx \ln(g)$

D $\ln(\pi) \approx \sqrt{\ln g}$

Question 4

A circle in the x-y plane has radius r, perimeter P, and area A. Which of these statements is true?

A $r > 2$ is a necessary but not sufficient condition for $P < A$

B $r > 0$ is a necessary and sufficient condition for $P < A$

C $0 < r < 2$ is a sufficient but not necessary condition for $P > A$

D $r > 2$ is a necessary and sufficient condition for $P > A$

E $0 < r < 2$ is a necessary and sufficient condition for $P < A$

F $r > 2$ is a sufficient but not necessary condition for $P < A$

G $0 < r < 2$ is a necessary and sufficient condition for $P > A$

Question 5

Let $I(a) = \int_0^a 2x - 3\,\mathrm{d}x$. For what values of a are $I(a)$ equal to -2?

A $a = 1, a = 2$

B $a = 0, a = 2$

C $a = 0, a = 1$

D $a = -2, a = -1$

E $a = -1, a = 2$

F $a = 0.5, a = 1$

G $a = -0.5, a = 1$

H $a = -2, a = 1$

Question 6

According to One Direction, you are beautiful iff you do not know you are beautiful. Also, if you flip your hair, One Direction will feel overwhelmed. Which of these statements must be true?

A If One Direction has felt overwhelmed, you have flipped your hair.

B If you do not know you are beautiful, it is possible that you are not beautiful.

C It is not possible for you to be beautiful.

D If One Direction has never felt overwhelmed, you have not flipped your hair.

E If you have flipped your hair or you do not know that you are beautiful, you are beautiful and One Direction has felt overwhelmed.

F It is not possible for you to have overwhelmed One Direction, but not be beautiful.

Question 7

What is the limit of $\frac{(x^2+5x+6)(x-1)}{(x+3)(x^2-1)}$ as $x \to \infty$?

A 0

B 1

C -1

D 2

E -2

F ∞

G $-\infty$

H Undefined

Question 8

The line $y = x$ is tangent to the curve $y = ax^2 + 5x + 1$ at the point with what x-coordinate?

A -0.5

B 5

C -1

D 1

E -2

F 2

Question 9

Evaluate $\sum_{r=0}^{\infty} \left(\frac{3}{10^r} + \frac{1}{10^{r+1}} + \frac{4}{10^{r+2}} \right)$.

A 3.14

B $\frac{3.14}{0.9}$

C 31.4

D $\frac{3.14}{9}$

E $\frac{3.14}{0.09}$

F $\frac{0.9}{3.14}$

G $\frac{3.14}{1.1}$

H $\frac{1.1}{3.14}$

Question 10

A coffee shop's 8 workers: Emily, Ethan, Olajide, Pranav, Rujul, Siddhant, Simon and Sabrina are all wearing T-shirts with the first of letters of their names printed on the fronts. How many different arrangements of the 8 workers causes their T-shirts to spell 'ESPRESSO' from left to right?

A 1

B 2

C 4

D 6

E 8

F 12

G 16

H 24

Question 11

Which function $f(x)$ satisfies the equation $\sqrt[4]{abcd} = f^{-1}\left(\frac{f(a)+f(b)+f(c)+f(d)}{4}\right)$, where $f^{-1}(x)$ is the inverse function of $f(x)$?

A $f(x) = x$

B $f(x) = x^2$

C $f(x) = \log_{2025}(x)$

D $f(x) = (\ln(x))^2$

E $f(x) = \ln(x^2)$

F $f(x) = e^x$

Question 12

Which line in this false proof that $1 = 2$ contains the first mistake?

$$\begin{array}{ll} \text{Let } a = b & (1) \\ a^2 = ab & (2) \\ a^2 - b^2 = ab - b^2 & (3) \\ (a+b)(a-b) = b(a-b) & (4) \\ a + b = b & (5) \\ a + a = a & (6) \\ 1 = 2 & (7) \end{array}$$

A (1)

B (2)

C (3)

D (4)

E (5)

F (6)

G (7)

Question 13

The real numbers a, b are such that $a > b$. You are given that the function $f(x)$ is defined over positive real x and is either always increasing or always decreasing. Which of these conditions is sufficient but not necessary for $f(a) < f(b)$ to be true?

A $f'(x) < 0$

B $f(x)$ is a decreasing function

C $f(x) = \log_c x, 0 < c < 1$

D $f(x) = \log_c x, c > 1$

Question 14

I promise to do at least 5 STEP 1 questions, at least 3 STEP II questions, and at least 1 STEP III question next week. If I break my promise, which of the following statements must be true?

A I will have done fewer than 5 STEP I questions, fewer than 3 STEP II questions, and no STEP III questions

B I will have done fewer than 6 STEP I questions, fewer than 4 STEP II questions, or fewer than 2 STEP III questions

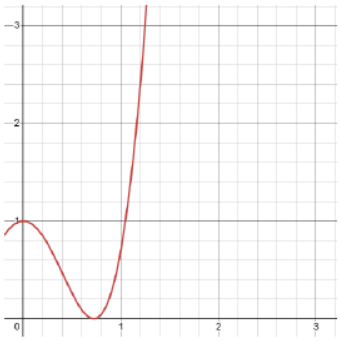
C I will have done fewer than 4 STEP I questions, fewer than 2 STEP II questions, and no STEP III questions

D I will have done fewer than 4 STEP I questions, fewer than 2 STEP II questions, or no STEP III questions

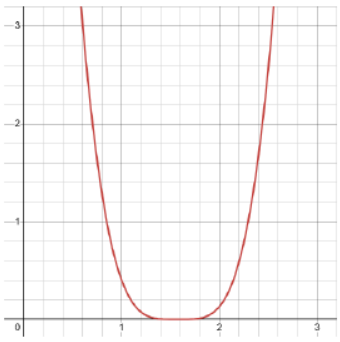
Question 15

Which of the following is a sketch of the graph $y = (3 - 4\sin x)^2$?

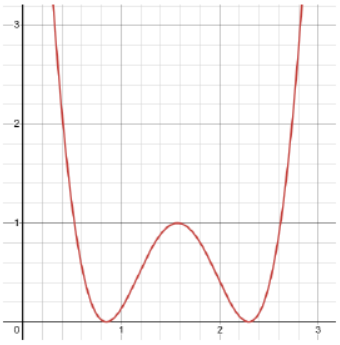
A



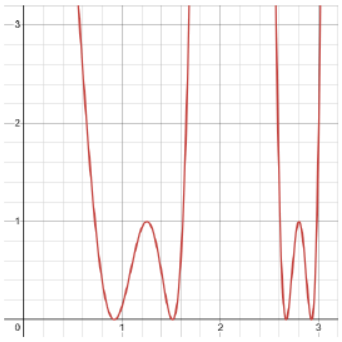
B



C



D



A A

B B

C C

D D

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p034-q015.

Question 16

The 5 Backstreet Boys are asked if they want it that way. Their responses are as follows:

Backstreet Boy A: “Exactly one of us wants it that way.” Backstreet Boy B: “Exactly two of us wants it that way.” Backstreet Boy C: “Exactly three of us wants it that way.” Backstreet Boy D: “Exactly four of us wants it that way.” Backstreet Boy E: “Exactly five of us wants it that way.”

If a Backstreet Boy does want it that way, they will tell the truth, but if a Backstreet Boy does not want it that way, they will lie. Which of these statements must be false?

A It is not necessary that at least one Backstreet Boy wants it that way.

B Backstreet Boy A is the only boy who can possibly want it that way.

C It is not possible that two or more of the Backstreet Boys want it that way.

D It may be possible that Backstreet Boy B wants it that way.

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p034-q016.

Question 17

A_n and B_n are an arithmetic sequence and a geometric sequence respectively, with first terms A_1 and B_1. You are given that $A_2 = B_3$ and $A_4 = B_2$, and that the common ratio of B_n is r. Which is a correct expression for A_1?

A $\frac{B_1 r(3r-1)}{2}$

B $\frac{B_1(3r^2-1)}{2}$

C $\frac{B_1(1-3r^2)}{2}$

D $\frac{B_1 r(1-3r)}{2}$

E $\frac{B_1 r^2(3r-1)}{2}$

F $\frac{B_1(3r-1)}{2}$

G $\frac{B_1 r^2(1-3r)}{2}$

H $\frac{B_1(1-3r)}{2}$

Question 18

For a given value of a, let $f(x) = x^{(x+a)^{(x+2a)^{(x+3a)^{\cdots}}}}$. What is the value of $f(0.3)$ when $a = 0.2$?

Note: $a^{b^{c^d}} = a^{\left(b^{\left(c^d\right)}\right)}$, i.e. power towers are evaluated from the top to the bottom.

A 0

B 0.7

C $\sqrt{0.3}$

D ∞

E 1

F 0.1

G $\sqrt{0.1}$

H 0.3

Question 19

Let $f(x)$ be the only degree 5 polynomial where the coefficient of x^5 is 1 and the following equations are true:

$$f(1) = 5$$
$$f(2) = 4$$
$$f(3) = 3$$
$$f(4) = 2$$
$$f(5) = 1$$

What is the value of $f(6)$?

A 0

B 6

C 30

D 36

E 72

F 120

Question 20

How many distinct positive integer pairs, (a, b), with $a > b$, solve the equation:

$$a^2 = b^2 + 2025?$$

Note: you are given that $2025 = 45^2 = 3^4 \times 5^2$.

A 0

B 1

C 2

D 3

E 5

F 7

G 35

H 105