TMUA

COMMUNITY PAPERS

Cody

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Zach/Cody

tmua.me editorial edition: corrections applied; original question numbering retained.
20 active questions.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

Question 1

How many pairs of positive integers (n, p), where p is prime, satisfy

$$n! + 5 = p?$$

A 0

B 2

C 4

D 3

E ∞

Question 2

In the figure below, the outside equilateral triangle contains infinitely many equilateral triangles, each with the same centre. The ratio of the side length of a triangle to the next inner triangle is k with $0 < k < 1$. The shaded area is $\frac{25}{34}$ of the area of the original triangle. Find k.



A $\frac{1}{2}$

B $\frac{4}{7}$

C $\frac{5}{7}$

D $\frac{3}{5}$

E $\frac{3}{4}$

Question 3

The roots of $x^3 + kx + p = 0$ are a, b, c. Compute

$$a^3b^4 + a^4b^3 + a^3c^4 + a^4c^3 + b^3c^4 + b^4c^3.$$

A pk^2

B $-pk^2$

C 0

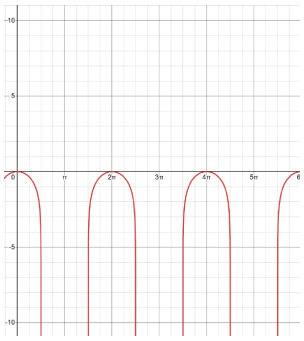
D $-p$

E k^3

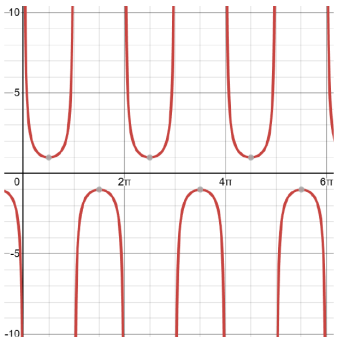
Question 4

Which of the following is the graph of $y = \ln(\sin x)$?

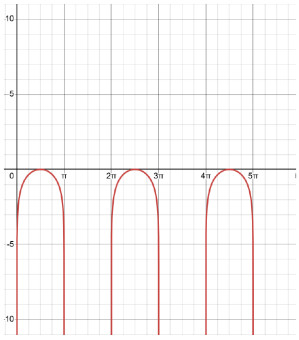
A A



B B



C C



Question 5

Find the area of the region satisfying

$$4 \leq x^2 + y^2 \leq 9, \qquad y^2 \geq 3x^2.$$

A $\frac{3\pi}{5}$

B 3π

C $\frac{4\pi}{9}$

D $\frac{5\pi}{3}$

E $\frac{8\pi}{5}$

Question 6

A sequence is defined by

$$x_{n+1} = \frac{1 - 2x_n\sqrt{x_n}}{3}, \qquad 0 < x_n < \left(\frac{1}{2}\right)^{2/3}.$$

If $x_n \to a$, find a.

A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{6}{7}$

D $\frac{1}{2}$

E $\frac{1}{4}$

Question 7

You roll four fair six-sided dice. What is the probability that the four numbers can form the sides of a rectangle?

A $\frac{5}{72}$

B $\frac{5}{54}$

C $\frac{1}{27}$

D $\frac{7}{54}$

E $\frac{2}{27}$

Question 8

Find the sum of all nonnegative integers n such that

$$16 + 2\binom{n}{1} + 4\binom{n}{2} + \cdots + 2^n\binom{n}{n} = 8\sqrt{3^n}.$$

A 8

B 2

C $2\log_3 5$

D $2\log_3 15$

E 0

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p096-q008.

Question 9

Find the remainder when

$$x^{12} + 7x^{10} + 9x^8 + 6x^6 + 5x^4 + 2x^2 + 8$$

is divided by $x^2 + 1$.

A 0

B 8

C $6 + 2x$

D $8 - 2x$

E -8

Question 10

The following are three consecutive terms of an arithmetic progression:

$$\log_3 2, \quad \log_3(2^x + 1), \quad \log_3(2^x + 10).$$

Find x.

A 1

B $\frac{\log_2 19}{2}$

C $\frac{\log_2 12}{3}$

D $\frac{\log_2 14}{2}$

E $\frac{\log_3 12}{3}$

Question 11

Let $f(x) = |x^3|$. What is $f'(x)$?

A $-3x|x|$

B $3x^2$

C $3x|x|$

D Undefined

E $3|x^2|$

Question 12

A sequence (a_n) satisfies, for every integer $n \geq 2$,

$$S_n = \sum_{k=0}^{n} a_k = \frac{n^2+3}{n-1}.$$

What is a_n as $n \to \infty$?

A 0

B 1

C 2

D $\frac{1}{2}$

E ∞

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p096-q012.

Question 13

Two football teams have 25 players combined. If Arithmetic FC sign Chris, their average age increases from 23 to 24. If Binomial United sign him, their average increases from 24 to 25. How old is Chris?

A 27

B 28

C 32

D 37

E 45

Question 14

Let

$$I = \int_0^a \left(2^{-x^3} - \frac{1}{2}\right) \, \mathrm{d}x.$$

Find a such that $\frac{dI}{da} = 0$.

A 1

B $\frac{1}{2}$

C 2

D $\frac{1}{4}$

E 0

Question 15

Let

$$f(x) = x^3 - \frac{3}{2}x^2 - 6x + k.$$

Find all k such that f has three real roots.

A $-\frac{7}{2} < k < 10$

B $0 < k < 10$

C $k < 0$ or $k > 10$

D $k < -\frac{7}{2}$ or $k > 10$

E All real values of k

Question 16

Find the area between the curves

$$\sin(x^2 + y^2) = 1, \qquad \sin(x^2 + y^2) = -1,$$

for $0 \leq x^2 + y^2 \leq 2\pi$.

A π

B $\frac{\pi^2}{4}$

C $\frac{\pi^2}{2}$

D π^2

E $\frac{2\pi}{5}$

Question 17

The three sides of a right triangle form the first three terms of an arithmetic sequence with positive common difference. The area is 18. Find the sum of the first 100 terms.

A $3\sqrt{3}$

B $\sqrt{3}$

C $5250\sqrt{3}$

D 1200

E 1800

Question 18

How many positive integer pairs (x, y) satisfy

$$x^3 + 9x^2y + 27xy^2 + 27y^3 = 3^{18}?$$

A 0

B 3^6

C $3^6 - 1$

D 3^5

E $3^5 - 1$

Question 19

Find the greatest integer n such that 3^n divides 100!.

A 44

B 45

C 46

D 47

E 48

Question 20

Which of the following is largest?

A $\left(\frac{\pi+e}{2}\right)^2$

B e^{π}

C π^e

D $\frac{\pi^3}{e}$