Beyond Horizon TMUA Mock Paper

Set 4 Paper 1

75 Minutes

20 Questions

One answer per question

tmua.me editorial edition: corrections applied; original question numbering retained.
20 active questions.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

1) Find the area bounded by the straight lines $x = -1$ and $x = 1$ and the graphs of $f(x)$ and $g(x)$, where $f(x) = x^3$ and

$$g(x) = \begin{cases} x^5 & \text{if } -1 \leq x \leq 0, \\ x & \text{if } 0 \leq x \leq 1, \end{cases}$$

(A) $\frac{1}{3}$

(B) $\frac{1}{8}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

(E) $\frac{1}{5}$

(F) $\frac{1}{6}$

2) The largest volume of a cube that can be enclosed in a sphere of diameter 2 cm, is, in cm^3,

(A) 1

(B) $2\sqrt{2}$

(C) π

(D) $\frac{8}{3\sqrt{3}}$

(E) $\frac{8}{9}$

(F) $\frac{2\sqrt{2}}{3}$

3) If α, β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are α^2, β^2 is

(A) $a^2x^2 + (b^2 - 2ac)x + c^2 = 0$

(B) $a^2x^2 - (b^2 + 2ac)x + c^2 = 0$

(C) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(D) $a^2x^2 - (a^2 - 2bc)x + c^2 = 0$

(E) $b^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(F) none of the above equations

4) Evaluate the following sum

$$(1 \cdot 1!) + (2 \cdot 2!) + (3 \cdot 3!) + \cdots + (50 \cdot 50!)$$

(A) 51!

(B) 2.5!

(C) 51! - 1

(D) 51! + 1

(E) 50! + 1

(F) 50! + 1

5) The number of distinct points common to the curves $x^2 + 4y^2 = 1$ and $4x^2 + y^2 = 4$ is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

6) In a parallelogram $PQRS$, P is the point $(-1,-1)$, Q is $(8,0)$ and R is $(7,5)$. Then S is the point

(A) $(-1,4)$

(B) $(-2,4)$

(C) $(-2,\frac{7}{2})$

(D) $(-\frac{3}{2},4)$

(E) $(-2,5)$

(F) $(-1,5)$

7) How many real solutions are there to the equation

$$2\tan^4 x - 5\tan^2 x + 3 = 0$$

in the interval $0 \leq x < 2\pi$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

(H) 7

(I) 8

8) Given that

$$\frac{dy}{dx} = 4x^2 - \frac{3-2x}{x^3}, \quad x \neq 0$$

and $y = 6$ when $x = 1$, find y in terms of x.

(A) $y = \frac{4}{3}x^3 + \frac{3}{2}x^{-2} - \frac{2}{3}x^{-1} + \frac{31}{6}$

(B) $y = 2x^3 + x^{-2} - x^{-1} + 5$

(C) $y = \frac{4}{3}x^3 + \frac{3}{2}x^{-2} - 2x^{-3} + \frac{23}{6}$

(D) $y = \frac{4}{3}x^3 + \frac{3}{2}x^{-2} - 2x^{-1} + \frac{31}{6}$

(E) $y = \frac{4}{3}x^3 + \frac{3}{2}x^{-2} - \frac{2}{3}x^{-3} + 4$

(F) $y = 3x^3 + x^{-2} - x^{-1} + 3$

9) Evaluate

$$\sum_{n=1}^{50} \log_{10}\left(2^{1-n}\right)$$

(A) $-495 \log_{10} 2$

(B) $495 \log_{10} 2$

(C) $-1225 \log_{10} 2$

(D) $1225 \log_{10} 2$

(E) $1 - 495 \log_{10} 2$

(F) $1 + 495 \log_{10} 2$

(G) $1 - 1225 \log_{10} 2$

(H) $1 + 1225 \log_{10} 2$

10) In the expansion of $(a+bx)^4$ the coefficient of x^3 is 4 times the coefficient of x^2. Given that a and b are non-zero positive integers, what is the smallest possible value of $a+b$?

(A) 5

(B) 6

(C) 7

(D) 11

(E) 15

(F) 19

Question 11

The functions f and g are given by $f(x) = 4x^2 + 10x + 5$ and $g(x) = x^3 + 5x^2 + 8x - 10$.

What is the complete set of values of x for which one of the functions is increasing and the other decreasing?

Here increasing and decreasing mean that the derivatives at the point are strictly positive and strictly negative, respectively.

A $x < -2 \quad \text{or} \quad -\frac{4}{3} < x < -\frac{5}{4}$

B $x \leq -2$

C $-4 \leq x \leq -3, x \geq -2$

D $x \leq -3, x \geq -2$

E $x \leq -4, -3 \leq x \leq -2$

F $x \leq -4, x \geq -3$

G $-3 \leq x \leq -2$

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p007-q011.

12) Find the value of

$$\int_1^5 \frac{4-3x}{x\sqrt{x}}\,dx$$

(A) $\frac{14\sqrt{5}-38}{\sqrt{5}}$

(B) $-\frac{95}{16}$

(C) $\frac{15\sqrt{5}-38}{\sqrt{5}}$

(D) -2

(E) $-\frac{1}{2}$

(F) $\frac{9}{4}$

(G) $\frac{14\sqrt{5}-38}{\sqrt{3}}$

13) Find the number of solutions of the equation

$$x \sin 3x = \cos x$$

in the interval $0 \leq x \leq 3\pi$.

(A) 1

(B) 3

(C) 5

(D) 7

(E) 9

14) The number of ways in which three non-negative integers n_1, n_2, n_3 can be chosen such that $n_1 + n_2 + n_3 = 10$ is

(A) 66

(B) 55

(C) 10^3

(D) $\frac{10!}{3!2!1!}$

(E) $\frac{9!}{3!2!1!}$

(F) 10^2

15) Find the complete set of values of the constant c for which the cubic equation

$$x^3 - 3x + c = 0$$

has three distinct real solutions.

(A) $0 < c < 2$

(B) $-2 < c < 0$

(C) $-2 < c < 2$

(D) $c > -2$

(E) $c < 2$

(F) $c < 0$

16) In the interval $0 \leq x \leq 2\pi$, the equation

$$\sin(3\cos(3x) + 1) = 0$$

has exactly

(A) 2 solutions

(B) 4 solutions

(C) 6 solutions

(D) 8 solutions

(E) 10 solutions

(F) 12 solutions

17) An equilateral triangle has centre O and side length 2. A straight line through O intersects the triangle at two distinct points P and Q. The minimum possible length of PQ is

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2\sqrt{3}}{3}$

(D) $\frac{4}{3}$

(E) $\sqrt{3}$

(F) $\sqrt{2}$

18) What is the area of the polygon formed by all the points (x, y) in the plane satisfying the inequality

$$|x - 2| + |y - 2| \leq 8?$$

(A) 32

(B) 64

(C) 96

(D) 128

(E) 160

19) Let A be a point on the circle with the equation $x^2 + y^2 = 4$, and let B be a point on the circle with the equation $x^2 + y^2 = 9$. If the angle between the vectors representing the points A and B is $30°$, what is the distance between the points A and B?

(A) $\sqrt{13}$

(B) $6\sqrt{3}$

(C) $\sqrt{3}$

(D) $\sqrt{13 - 6\sqrt{3}}$

(E) $\sqrt{13 + 6\sqrt{3}}$

20) What is the degree of the following polynomial

$$\frac{d^2}{dx^2}\left[(3x-2)^4(2-x)^5\right]+\frac{d}{dx}\left[(3x+2)^4(4x^2-3)^2\right]$$

(A) 11

(B) 10

(C) 9

(D) 8

(E) 7

(F) less than 7