Beyond Horizon TMUA Mock Paper

Set 1 Paper 1

75 Minutes

20 Questions

One answer per question

tmua.me editorial edition: corrections applied; original question numbering retained.
19 active questions. Withdrawn question(s): 20.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

1) For what values of x is $f(x)$ a valid function?

$$f(x) = \frac{1}{2\log(x^2 - 2x - 3)}$$

(A) $(0, \infty)$

(B) $(-\infty, -1) \quad x \neq 1 \pm \sqrt{5}$

(C) $(-\infty, -1) \cup (3, \infty) \quad x \neq 1 \pm \sqrt{5}$

(D) $(-\infty, -3) \cup (1, \infty) \quad x \neq 1 \pm \sqrt{5}$

(E) $f(x)$ is never valid

(F) $(-\infty, -1) \cup (3, \infty)$

2) For a real number x, let $[x]$ denote the largest integer less than or equal to x. Find the value of

$$\int_{-100}^{100} [t^3]\, dt$$

(A) 0

(B) 100

(C) -100^2

(D) -100^3

(E) 100^2

(F) -100

3) Two functions are defined as:

$$f(x) = \left|x^2 - 4\right|, x \in \mathbb{R}$$

$$g(x) = x + 2, x \in \mathbb{R}$$

Which range of values satisfy the inequality $f(x) < g(x)$?

(A) $-2 < x < 3$

(B) $-2 < x < 1$

(C) $1 < x < 3$

(D) $x < 1$ and $x > 3$

(E) $x < -2$ and $x > 3$

(F) $x < -2$ and $x > 1$

4) What is the sum of the coefficients in $\left(\frac{2}{x} + \frac{3}{x^2} + 5x\right)^3$?

(A) 1000

(B) 750

(C) 125

(D) 343

(E) 100

(F) 729

5) How many solutions does this equation have in the interval $0 \leq x \leq 2\pi$?

$$\sin(\cos(\sin(x))) = \cos(\sin(\cos(x)))$$

(A) 0

(B) 1

(C) 2

(D) 4

(E) 8

(F) 16

6) How many integer pairs (x, y) satisfy the equation

$$x^2 + 6x + y^2 = 4$$

(A) 2

(B) 4

(C) 6

(D) 8

(E) 10

(F) 12

7) The number of polynomials of the form x^3+ax^2+bx+c which are divisible by x^2+1 and where a, b, and c belong to $\{1, 2, \ldots, 10\}$, is

(A) 1

(B) 10

(C) 11

(D) 12

(E) 100

(F) 101

8) There are seven greeting cards, each of a different colour, and seven envelopes of the same seven colours. The number of ways in which the cards can be put in the envelopes, so that exactly four of the cards go into the envelopes of the right colours, is

(A) $\binom{7}{3}$

(B) $2\binom{7}{3}$

(C) $(3!)\binom{4}{3}$

(D) $(3!)\binom{7}{3}\binom{4}{3}$

(E) $(3!)\binom{7}{3}$

(F) $(4!)\binom{7}{3}$

9) Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, 3, \ldots$ and $F(1) = 2$. Then $F(101)$ equals

(A) 50

(B) 52

(C) 54

(D) 56

(E) 58

(F) None of the above

10) Let $p(x)$ be a continuous function which is positive for all x and

$$\int_2^3 p(x)\,dx = c\int_0^2 p\left(\frac{x+4}{2}\right)\,dx.$$

Then

(A) $c = 4$

(B) $c = -\frac{1}{4}$

(C) $c = \frac{1}{4}$

(D) $c = 2$

(E) $c = -\frac{1}{2}$

(F) $c = \frac{1}{2}$

11) How many real solutions does the equation $2\sin^3(x) - 6\sin(x)\cos^2(x) + \sin(x) = 0$ have in the interval $0 < x < 2\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

12) Suppose $a < b$. The maximum value of the integral

$$\int_a^b \left(\frac{3}{4} - x - x^2\right) dx$$

over all possible values of a and b is

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) $\frac{2}{3}$

(E) $\frac{4}{5}$

(F) $\frac{5}{4}$

13) The line $x = y$ is tangent at $(0, 0)$ to a circle of radius 1. The centre of the circle is

(A) $(1, 0)$

(B) either $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(C) either $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(D) $(0, 1)$

(E) $(0, 0)$

(F) None of the Above

14) The mth term of an arithmetic progression is x and the nth term is y. What is the sum of the first $(m+n)$ terms?

(A) $\frac{m+n}{2}\left[(x+y)+\frac{x-y}{m-n}\right]$

(B) $\frac{m+n}{2}\left[(x-y)+\frac{x+y}{m-n}\right]$

(C) $\frac{1}{2}\left[\frac{x+y}{m+n}+\frac{x-y}{m-n}\right]$

(D) $\frac{1}{2}\left[\frac{x+y}{m+n}-\frac{x-y}{m-n}\right]$

(E) $\frac{m-n}{2}\left[(x-y)+\frac{x+y}{m-n}\right]$

(F) $\frac{m+n}{2}\left[(x+y)+\frac{x+y}{m-n}\right]$

15) The values of m for which $mx^2 - 6mx + 5m + 1 > 0$ for all real x is

(A) $0 < m < \frac{1}{4}$

(B) $0 \leq m < \frac{1}{8}$

(C) $m > 0$

(D) $0 \leq m < \frac{1}{4}$

(E) $m < 0$

(F) $m = 0$

16) The inequality $\sqrt{x+6} \geq x$ is satisfied for real x if and only if

(A) $-3 \leq x \leq 3$

(B) $-2 \leq x \leq 3$

(C) $-6 \leq x \leq 3$

(D) $0 \leq x \leq 6$

(E) $-6 < x < 6$

(F) $0 < x < 3$

17) The difference between the roots of the equation $6x^2 + \alpha x + 1 = 0$ is $\frac{1}{6}$. α is a positive number. The value of α is

(A) 2

(B) 3

(C) 4

(D) 5

(E) $\frac{8}{3}$

(F) 8

18) Let x, y, z be positive. The least value of the following expression is

$$\frac{x(1+y)+y(1+z)+z(1+x)}{\sqrt{xyz}}$$

(A) $\frac{9}{\sqrt{2}}$

(B) 6

(C) $\frac{1}{\sqrt{6}}$

(D) 3

(E) 1

(F) None of the Above

19) All the letters of the word PESSIMISTIC are to be arranged so that no two S's, no two I's, and S and I do not occur together. The number of such arrangements is

(A) 1800

(B) 5480

(C) 4800

(D) 1200

(E) 2400

(F) 1801