TMUA

COMMUNITY PAPERS

Asher Falcon TMUA Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Asher Falcon

tmua.me editorial edition: corrections applied; original question numbering retained.
20 active questions.
Credit remains with the original author. Editorial history: model-answers/ALTERED-QUESTIONS.md in the repository.

Question 1

What is the value of

$$\sqrt{156 + \sqrt{156 + \sqrt{156 + \cdots}}}?$$

A 8

B $2\sqrt{39}$

C 12

D 13

E $\frac{159}{12}$

F $\frac{170}{13}$

Question 2

If $f(\sin(x)) = 0$ has a solution at $x = \frac{\pi}{2}$, which of the following x could be a solution to

$$2f(2\sin(x)) = 0$$

A π

B $\frac{\pi}{2}$

C $\frac{\pi}{3}$

D $\frac{\pi}{4}$

E $\frac{\pi}{6}$

F $\frac{3\pi}{2}$

G $\frac{7\pi}{6}$

H $\frac{5\pi}{4}$

I There is no such x

Question 3

How many solutions are there to

$$|\sin(x^2)| - |\cos(x^2)| = \frac{\pi}{4}$$

in the range $0 \leq x \leq \pi$?

A 0

B 1

C 2

D 3

E 4

F 5

G 6

H 7

I 8

Question 4

What is the value of

$$\frac{1}{67} + \frac{11}{67^2} + \frac{111}{67^3} + \frac{1111}{67^4} + \cdots ?$$

A $\frac{\sqrt{67}}{67}$

B $\frac{11}{67}$

C $\frac{11}{627}$

D $\frac{67}{3762}$

E $\frac{67}{418}$

F $\frac{7}{396}$

G $\frac{2}{67}$

H $\frac{5}{1881}$

Question 5

The coefficient of $x^3y^4z^3$ in the expansion of

$$(2x + 4y^2 + 8z^3 + 128)^8$$

can be expressed in the form $a \cdot 2^b$. What is $a + b$?

A 24

B 25

C 34

D 42

E 52

F 56

G 127

H 128

I 130

J 133

K 865

Question 6

The function $f(x)$ is defined as

$$f(x) = 4.\dot{3}\dot{5}x$$

What is the value, in its simplest form, of

$$\int_1^2 f(x)\,\mathrm{d}x$$

A $\frac{13}{2}$

B $\frac{72}{11}$

C $\frac{261}{40}$

D $\frac{431}{66}$

E $\frac{862}{99}$

F $\frac{431}{99}$

G $\frac{1293}{198}$

H $\frac{1724}{198}$

I $\frac{862}{198}$

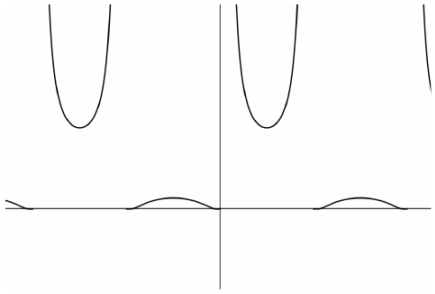
J $\frac{431}{198}$

Question 7

Which of the following graphs shows

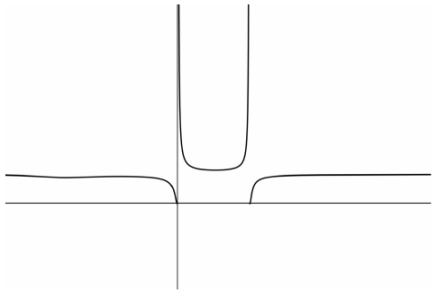
$$e^{\frac{1}{\sin(x)}}$$

A A



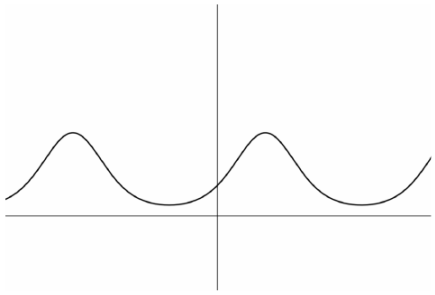
A

B B



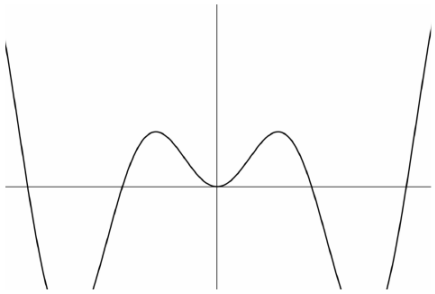
B

C C



C

D D



D

Question 8

Which has the smallest value

A $\frac{e^{\pi}}{\pi}$

B $\frac{\pi^{e}}{e}$

C $\frac{\tan(\frac{\pi}{3})}{3}$

D $\frac{2.\dot{2}}{3.\dot{3}}$

E $\sin(0.75\pi)$

F $\frac{162}{270}$

Question 9

What is the area of the region satisfied by the two inequalities

$$21 + 3y^2 \leq -3x(x + 8)$$

$$y \leq -|x + 4|$$

A π

B 9π

C $\frac{3\pi}{4}$

D $\frac{9\pi}{2}$

E $\frac{9\pi}{4}$

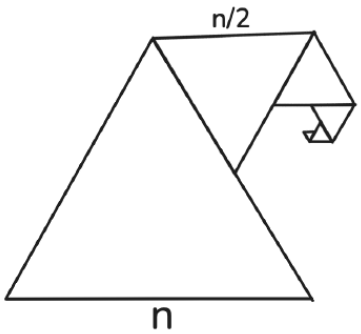
F $\frac{9}{\sqrt{2}}$

Question 10

Let $f(n)$ be defined as the total area of all the equilateral triangles starting with side length n and then being successively halved.

What is the value of

$$\int_0^2 f(n)\,\mathrm{d}n$$



A $\frac{2\sqrt{3}}{9}$

B $\frac{5\sqrt{3}}{9}$

C $\frac{8}{3}$

D $\frac{8}{9}$

E $\frac{24\sqrt{3}}{7}$

F $\frac{8\sqrt{3}}{9}$

G $\frac{8\sqrt{3}}{3}$

H $\frac{5\sqrt{3}}{3}$

I $\sqrt{3}$

Question 11

What is the product of the two values of b which satisfy

$$2(\log_a b)^2 + \frac{\ln(b^{10})}{\ln(a^2)} - 11 = \log_a(b^2)\log_b(\sqrt{a})?$$

A $a^{-5/2}$

B $a^{-1/2}$

C $a^{3/2}$

D a^4

E a^6

F 1

G 0

Question 12

$$ax^5 + 2x^5 + bx^3 + 4x^2 + 5x + 8$$

Given that the function above has a factor of $(x^2 + 2)$, what is the value of $2a - b$?

A -13

B 13

C $\frac{13}{2}$

D $-\frac{13}{2}$

E $\frac{5}{2}$

F $-\frac{5}{2}$

G 4

H -4

I The answer cannot be identified

Question 13

Let $F(x)$ be the smallest prime factor of x, for example $F(9) = 3$ and $F(125) = 5$.

$$F\left(67^{2048} - 1\right) =$$

A 1

B 2

C 3

D 7

E 67

Question 14

For which function is $f(f(f(x))) > 0$ for all real x?

A $f(x) = \sin(x)$

B $f(x) = |x|$

C $f(x) = x^3$

D $f(x) = \cos(x)$

E $f(x) = \ln(x)$

F $f(x) = \tan(x)$

G $f(x) = \sin^2(x)$

Question 15

What is the range of c for which $\sqrt{x+4} = x + c$ has exactly two real solutions?

A $-4 \leq c \leq 4$

B $-4 < c < 4$

C $4 \leq c < \frac{19}{4}$

D $c \leq \frac{19}{4}$

E $-4 < c \leq \frac{19}{4}$

F $c < \frac{17}{4}$

G $-4 < c < \frac{17}{4}$

H $4 \leq c \leq \frac{17}{4}$

I $4 \leq c < \frac{17}{4}$

J $4 < c < \frac{17}{4}$

Question 16

What is the product of the real roots of the equation

$$\ln(x)^2 \left(\ln\left(x^6\right) - 7\right) + \ln\left(\frac{e^{14}}{x^{12}}\right) = 0$$

A $e^{2\sqrt{2}}$

B 1

C $e^{6/7}$

D $e^{7/6}$

E $e^{-7/3}$

F $e^{7\sqrt{2}/6}$

Question 17

A function satisfies $f(f(x)) = 9x - 20$. What is the possible positive value of $f(9)$?

A 61

B 22

C 7

D 27

E 81

F 9

Question 18

A student defines the function $g(x) = x^2 - 3x + 2$, and then makes a new function:

$$G(x) = \frac{g^{15}(x)}{g^{15}(x-1)}$$

What is $G(5)$?

A 2^{30}

B 2^{29}

C 2^{28}

D 2^{25}

E 2^{24}

F 2^{16}

G 2^{15}

Question 19

What is the last digit of

$$3^{123} \cdot 7^{456} + 1?$$

A 0

B 1

C 2

D 3

E 4

F 5

G 6

H 7

I 8

J 9

Question 20

Find the total area of all bounded regions formed by the x-axis, the line $x = 1$, and the curve

$$y = 3x|x + x^2|.$$

A $\frac{3}{2}$

B $\frac{1}{2}$

C $-\frac{1}{2}$

D $\frac{3}{4}$

E $\frac{5}{4}$

F 2

G $-\frac{3}{4}$

H -2

I $\frac{7}{4}$

J $\frac{5}{2}$

This page contains an editorial correction to the supplied paper. The source paper is credited to its original author. Question IDs on this page: p095-q020.